Collusion-Enhancing Institutions in Dispersed Oligopolies*

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Abstract

Collusion in oligopoly is a fit subject for analysing how institutions help coordinate Pareto-improving social behavior. This notwithstanding, economic analysis has devoted little attention to the institutional underpinnings of collusion behavior. Deliberate institutional design is particularly needed when the number of firms is large since, under such conditions, factors that facilitate collusion must be manipulated through artificial arrangements, to overcoming the "critical discount rate" becoming smaller as the number of firms increases. In this paper we claim that a typical institutional arrangement to sustain collusion in dispersed industries calls for an artificial control of the firms' marginal cost function. First, we provide a novel result to show that, in a dynamic Cournot model, under decreasing returns, collusion can always be sustained in equilibrium for any given discount factor provided the marginal cost function is sufficiently steep. Moreover, as the number of firms increases, the aggregate collusive profits remain bounded away from zero and the degree of collusion remains constant and strictly greater than one. Second, we provide evidence of collusion-enhancing institutions of this kind in dispersed industries and discuss how the results obtained can improve our understanding of "facilitating practices" in antitrust analysis.

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1 Introduction

It is standard view to link the likelihood of collusive behavior to a 'structural' index, such as the number of firms in the industry. For instance, Scherer and Ross (1990, p. 277) suggest "*as a very crude and general rule*" that, "*if evenly matched firms supply homogeneous products in a well-defined market, they are likely to begin ignoring their influence on price when their number exceeds ten or twelve*". The structural presumption dates back to Adam Smith and the English classical school (see Viner, 1960), and is also well-rooted in the modern theory of industrial organization, as the analysis of the incentive compatibility constraint for collusion shows that the 'critical discount factor' always becomes smaller as the number of firms in the industry increases.[1] Accordingly, Motta (2004) lists the number of firms as the first, and perhaps the most important, among the "factors that facilitate collusion", though the author is careful in specifying that this holds "other things being equal".

Building on the structural view, in antitrust case law, a concentrated oligopoly has been usually held as a necessary condition whenever an explicit hard-core agreement is absent and the antitrust agency has to evaluate the circumstantial evidence of a collusive behavior. Antitrust agencies and judges have typically followed this view when evaluating information exchanges.[2] In the *Container*

[1] This notwithstanding, Scherer and Ross' "crude rule" is unwarranted. For instance, under linear demand and cost functions, and for plausible values of the discount rate under the hypothesis of a one-year detection period ($0,025 \leq r \leq 0,05$), the maximum number of firms, n^*, for which the incentive compatibility constraint is satisfied ranges between respectively twenty-one and forty-one, under Bertrand competition – when the incentive compatibility constraint binds if $r = 1/(n^* - 1)$ – and seventy-seven and one hundred-fifty-seven under Cournot competition – when the incentive compatibility constraint binds if $r = 4n^*/(n^* + 1)^2$. Note that the numbers above are likely to be an underestimate of the maximum number of colluding firms, since in most industries the actual time needed for the other firms' reaction is plausibly shorter than one year, implying smaller values of the discount rate than those assumed here.

[2] For a general economic assessment of information exchanges in the antitrust perspective,

Corporation decision (1969) the U.S. Supreme Court found the exchange of information among competitors in a highly concentrated industry to be unlawful.[3] In the decision *UK Tractors* (1992), the European Commission explicitly took account of the high concentration in the market of agricultural tractors in the UK.[4] An even sharper view was expressed in the *Wirtschaftsvereinigung Stahl* decision (1998) whereby the Commission stated that "*...the assessment of the exchange is directly linked with the degree of concentration of the market...*".[5]

However, the conventional approach was challenged by the Italian Competition Authority in the case *RCA* (2000), where the defendants had pointed out that the Italian car insurance industry could not be defined as a 'concentrated oligopoly' in accordance with the *UK Tractors* decision, since a 'large' number of firms (namely forty-four) had been involved in the contested information exchange agreement. In contrast, the Italian Competition Authority argued that the information agreement had an anticompetitive 'object' precisely because it relaxed the incentive constraint which, under the structural condition of 'dispersed oligopoly', would prevent collusive behavior to arise as a noncompetitive (Nash) equilibrium in the indefinitely repeated game.

The *RCA* decision is consistent with the analysis of collusion in oligopoly. Regardless of the number of firms, in fact, collusion can always be sustained if

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see Kühn (2001).

[3] The U.S. approach to competitor communications is thoroughly reviewed by DeSanti and Nagata (1994).

[4] See European Commission, *UK Agricultural Tractor Registration Exchange*, 1992. The eight companies that participated in the agreement held 88% of the UK tractor market. The first four companies shared 77% of the market, 80% after Ford New Holland was taken over by Fiat.

[5] The Tribunal of First Instance and the European Court of Justice confirmed the Commission's approach in their judgements concerning the *UK Tractors* case (see Tribunal of First Instance, Judgements of 27 October 1994 in case T-35/92 *John Deere* and T-34/92 *Fiatagri and Ford New Holland*; European Court of Justice, Cases C-7/95 *John Deere Ltd. V. EC Commission*, 1998). More recently, the European Court of Justice has accepted the standard view also in the decision *Asnef-Equifax/Asociación de Usuarios de Servicios Bancarios* (2006).

the discount factor, $\delta = 1/(1+r)$, where r is the discount rate for a conventional 'one-period', is sufficiently large, i.e. close to one. An information exchange reduces the time interval for firms' reaction to a deviation from a collusive behavior. For any given instantaneous discount rate, a shorter time interval for firms' reaction to a deviation from a collusive behavior implies a smaller value of the one-period discount rate, and a correspondingly larger value of the discount factor δ – that converges to one as r goes to zero. Thus, provided the length of the reaction time can be made sufficiently short, collusion can be sustained in equilibrium even in dispersed oligopolies.

In this paper we analyze the sustainability of collusion in industries with a large number of firms in a novel perspective. We show that, in a dynamic Cournot model, there exist circumstances under which the incentive to collude does not vanish when the number of firms increases, and that those circumstances do not require the discount factor to become adequately close to one. More specifically, we focus on technology and prove that, under decreasing returns to scale, the incentive constraint for collusion can always be satisfied in an oligopoly, whatever the number of firms, for any given discount factor. In our setting, the crucial variable is the slope of the marginal cost function: we show that the steeper is the slope of the marginal cost function, the larger is the number of firms for which collusion can be sustained *ceteris paribus*. The basic intuition is that a steeper marginal cost function reduces the incentive for a firm to deviate from a collusive behavior, thus weakening the 'cheating effect'. We also prove that, when the number of firms increases, there always exists a value of the slope of the marginal cost function such that the degree of collusion (measured by the ratio of the collusive profits to the Cournot profits) remains constant and non-trivially greater than one for arbitrarily many firms. Moreover, the aggregate collusive profits in the industry and, correspondingly, the welfare loss induced by collusion both converge to a finite value bounded away from zero, when the number of firms grows indefinitely.

The analysis of the circumstances under which (tacit) collusion can be attained independently of the 'structure' of the market betters our understanding of how, in dispersed oligopolies, firms may exploit those conditions by means of several different arrangements, or 'facilitating practices' (see Grillo, 2002). The organization of an information exchange to 'control' the reaction time to a deviation is just one of such possible arrangements. In the perspective of this paper, we suggest that firms might also want to 'control' their marginal cost function with the purpose of raising its slope. We provide some evidence on how such a goal can be achieved in dispersed industries.

In a theoretical perspective, our analysis can be related to two strands of literature. The first is concerned with collusive behavior under capacity constraints (a limit case of decreasing returns). Brock and Scheinkman (1985) show that, when the firms in the industry are exogenously constrained in capacity, a non-monotonic effect on collusion arises as their number increases, due to the fact that the associated increase in total capacity with respect to the industry collusive output makes the threat of retaliation stronger. Benoit and Krishna (1987) and Davidson and Deneckere (1990) endogenize the choice of capacity and show that, in presence of excess capacity, the cheating effect is outweighed by the punishment effect.[6] More recently, Kühn (2006) has argued that the fragmentation of capacity facilitates collusion and increases the highest sustainable collusive price when individual firms are capacity constrained relative to total demand. The idea that, in the dynamic setting, strictly cost convexity weakens the cheating effect has also been exploited by Weibull (2006), who provides a generalization of Bertrand competition to convex cost functions.

The second strand of related literature concerns the sustainability of a non-trivial degree of collusion when the number of firms grows. In an indefinitely repeated game with free entry, MacLeod (1987) and Stiglitz (1987) show that the

[6] In Pénard (1997) and in Compte, Jenny and Rey (2002) asymmetry in capacities hurts collusion when aggregate capacity is limited with respect to the market size.

joint profit maximum can be sustained in equilibrium under Bertrand reversion, provided the discount factor becomes sufficiently close to one. The same result is obtained by Harrington (1989) under Abreu-type predatory retaliation, and by Harrington (1991) under Cournot reversion as long as the height of entry barriers is positive. Differently from the above results, we obtain a constant and non trivial degree of collusion for arbitrarily many firms independently of the value of the discount factor.

The paper is organized as follows. Section 2 develops a model of an industry with decreasing returns whereby, for any given discount factor, collusion can always be sustained regardless of the number of firms, provided the marginal cost function is sufficiently steep. Furthermore, it is shown that there always exist conditions under which the degree of collusion remains constant and non-trivial when the number of firms grows indefinitely. Section 3 discusses the implications of the theoretical model for competition policy.

2 The model

Consider an industry where a large number of identical firms i, $i \in \{1, ..., n\}$, produce a homogeneous good, using a technology with decreasing returns.

We assume a linear inverse market demand function

$$p = a - bQ, \tag{1}$$

$a > 0$, $b > 0$ and $Q = \sum_{i=1}^{n} q_i$, where Q denotes aggregate production and q_i is the quantity produced by the generic firm i.

We also assume that each firm faces a cost function of the type

$$C(q_i) = \alpha q_i^2, \tag{2}$$

with $\alpha > 0$.

To start with, we look for the existence of collusive equilibria in the industry when the indefinitely repeated game is built on the following trigger strategy: at stage zero each firm plays the collusion quantity q^M (as the game is symmetric q^M is equal to $1/n$-th of the quantity that a multi-plant monopolist, facing the cost function (2) at each plant, would produce) and then continues to play it provided that in the preceding stage all other firms have played the collusion quantity. If, in any period, a firm deviates from the collusive strategy — by playing a quantity q^D that is the best response to the other players producing q^M — a punishment phase in which all firms play the Cournot quantity q^C forever follows.

First, we calculate the quantity produced and the profits under collusion, deviation and punishment, respectively.

The collusion quantity q^M can be obtained as the solution of the following profit maximization problem

$$\max_{q_1,....,q_n} \quad \left(a - b\sum_{j=1}^{n} q_j\right)(q_1 + ... + q_n) - \alpha\left(q_1^2 + ...q_n^2\right). \tag{3}$$

As the inverse market demand function is linear and each cost function is strictly convex, first order conditions are necessary and sufficient for a maximum. From the first order condition each firm's collusive quantity is:

$$q_i^M \equiv q^M := \frac{a}{2(\alpha + nb)}, \tag{4}$$

which implies that the collusive price is

$$p^M = a\frac{2\alpha + nb}{2(\alpha + nb)} \tag{5}$$

and that each firm's profits under the collusive agreement are

$$\Pi_i^M = \frac{a^2}{4(\alpha + nb)}. \tag{6}$$

Observe that Π_i^M is strictly decreasing in n and $\partial\Pi_i^M/\partial\alpha < 0$.

The quantity q_i^D produced by firm i that deviates from the collusive equilibrium when the other $(n-1)$ firms stick to it can be obtained as the solution of the following problem

$$\max_{q_i} \quad \left(a - bq_i - \frac{ab\,(n-1)}{2\,(\alpha+nb)}\right) q_i - \alpha q_i^2. \tag{7}$$

From the first order condition and after some algebra

$$q_i^D = \frac{a\,(2\alpha+(n+1)\,b)}{4\,(\alpha+b)\,(\alpha+nb)} = q_i^M \cdot \frac{2\alpha+(n+1)\,b}{2\,(\alpha+b)}, \tag{8}$$

$$p^D = \frac{a\,(2\alpha+b)\,(2\alpha+(n+1)\,b)}{4\,(\alpha+b)\,(\alpha+nb)}, \tag{9}$$

and

$$\Pi_i^D = \frac{a^2\,(2\alpha+(n+1)\,b)^2}{16\,(\alpha+b)\,(\alpha+nb)^2}, \tag{10}$$

where Π_i^D are the deviation profits. Notice that Π_i^D is strictly decreasing in n, and $\partial\Pi_i^D/\partial\alpha < 0$. Notice also that Π_i^D can be written as

$$\Pi_i^D = \Pi_i^M \cdot H\,(n,\alpha)\,, \tag{11}$$

where

$$H\,(n,\alpha) := \frac{(2\alpha+b\,(n+1))^2}{4\,(\alpha+nb)\,(\alpha+b)}. \tag{12}$$

It is not difficult to show that, for $n > 1$, the following properties of $H\,(n,\alpha)$ hold:

1. $H\,(n,\alpha) > 1$ for all α;

2. $H\,(n,\alpha)$ is strictly increasing in n;

3. $\partial H\,(n,\alpha)\,/\partial\alpha < 0$;

4. $\lim\limits_{\alpha\to+\infty} H\,(n,\alpha) = 1$.

As Π_i^M is strictly decreasing in n and $\partial\Pi_i^M/\partial\alpha < 0$, the deviation profits Π_i^D decrease faster than Π_i^M when α increases given n, and slower than Π_i^M when n increases given α. By using the expression (8) for q_i^D it is also easy to see that the ratio between the quantity produced by the deviating firm and the one supplied under the collusive agreement is increasing in the number of firms in the market given α, while it converges to one as α increases given n.

In the (permanent) punishment phase following deviation, each firm reverts to the Nash equilibrium strategy of the constituent game, producing the Cournot quantity q_i^C, which can be obtained as the solution of the problem

$$\max_{q_i} \quad \left(a - bq_i - b\sum_{j\neq i} q_j\right) q_i - \alpha q_i^2, \tag{13}$$

implying that

$$q_i^C = \frac{a}{2\alpha + b(n+1)}, \tag{14}$$

$$p^C = a\frac{2\alpha + b}{2\alpha + b(n+1)} \tag{15}$$

and

$$\Pi_i^C = \frac{a^2(\alpha + b)}{(2\alpha + b(n+1))^2}, \tag{16}$$

where Π_i^C are the Cournot profits.[7]

Observe that $H(n,\alpha)\cdot\Pi_i^C = \Pi_i^M$. Hence:

Definition 1 *The degree of collusion in the industry is* $H(n,\alpha) := \Pi_i^M/\Pi_i^C$.

It is interesting to note that within the specific setting of this paper

$$H(n,\alpha) = \frac{\Pi_i^D}{\Pi_i^M} = \frac{\Pi_i^M}{\Pi_i^C}. \tag{17}$$

For a given discount factor δ, the collusive agreement can be sustained if and only if the *incentive compatibility constraint* (IC) faced by each firm is satisfied. Focusing again on firm i, IC can be written as

$$\frac{1}{1-\delta}\Pi_i^M \geq \Pi_i^D + \frac{\delta}{1-\delta}\Pi_i^C, \tag{18}$$

and, rearranging,

$$\delta \geq \frac{\Pi_i^D - \Pi_i^M}{\Pi_i^D - \Pi_i^C}.$$

[7]By comparing (6), (10) and (16) one can immediately see that, for $n > 1$ and for $\alpha > 0$, $\Pi_i^D > \Pi_i^M > \Pi_i^C$.

Recalling that $\delta = \frac{1}{1+r}$, where r denotes the discount rate, the above inequality can also be written as

$$r \leq \frac{\Pi_i^M - \Pi_i^C}{\Pi_i^D - \Pi_i^M}. \tag{19}$$

By exploiting (17), the incentive compatibility constraint (19) simply reduces to

$$H(n, \alpha) \cdot r \leq 1. \tag{20}$$

Notice that, as $H(n, \alpha) > 1$ for all α and $n > 1$, it straightforwardly follows from (20) that $r < 1$ is necessary for collusion.

The following proposition establishes that it is always possible to sustain collusion regardless of the number of firms for any $r < 1$.

Proposition 1

Let $r \in (0, 1)$. For all $n > 1$, define $\bar{H}(n) := H(n, 0)$ and consider the two cases: (A) $\bar{H}(n) \leq 1/r$ and (B) $\bar{H}(n) > 1/r$.

(A) If $\bar{H}(n) \leq 1/r$ then

1. $H(n, \alpha) \cdot r < 1$ for all $\alpha > 0$.

(B) If $\bar{H}(n) > 1/r$, then:

2. there exists a unique $\alpha^*(n) > 0$ such that $H(n, \alpha^*(n)) \cdot r = 1$, and, for all $\alpha \in [\alpha^*(n), +\infty)$ it is $H(n, \alpha) \cdot r \leq 1$;

3. $\alpha^*(n)$ is increasing in n.

Proof.

1 and 2. Recall that $\bar{H}(n) > 1$ for all $n > 1$ and $\lim_{\alpha \to +\infty} H(n, \alpha) = 1$. Both claims follow since $H(n, \alpha)$ is a continuous function monotonically decreasing in α.

3. Compute $\alpha^*(n)$ from the condition $H(n, \alpha^*(n)) \cdot r = 1$, i.e.

$$\alpha^*(n) := b[\mu(n-1) - 1],$$

where $\mu = \left((1-r)^{-1/2} - 1\right)/2 > 0$. The claim obtains by noting that $\alpha^*(n+1) - \alpha^*(n) = b\mu > 0$. ■

Discussion of Proposition 1. Proposition 1 shows that for any given discount rate, there always exists a cost parameter α which makes collusion sustainable for every n. Figure 1 qualitatively illustrates the Proposition. Provided n is

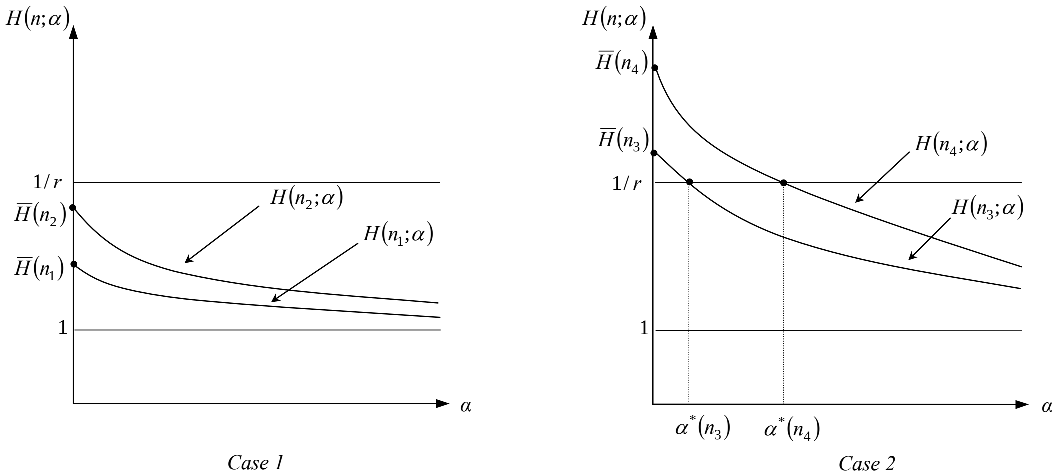

Figure 1: An illustration of Proposition 1, where $n_4 > n_3 > n_2 > n_1$

sufficiently low given r (*Case 1* in Figure 1), the incentive compatibility constraint is met for all α. For higher values of n (*Case 2* in Figure 1), there always exists a value of the cost parameter α, increasing in the number of firms, such that the collusive agreement is sustainable. The intuition behind the result is that an increase in the slope of the marginal cost function reduces the profits that a firm can achieve by deviating from the collusive agreement. As a firm contemplating a deviation from the collusive agreement trades-off the larger profits it can obtain in the deviation stage with the smaller present value of the profits under the punishment phase, vis-a-vis the present value of the profits it can achieve along the collusion path, it is always possible to find a slope of the marginal cost function that renders a deviation not incentive compatible.

Even more important than showing that collusion can be sustained with arbitrarily many firms for any given discount rate, it is to emphasize that the sustainable degree of collusion $H(n, \alpha)$ is always greater than one, and hence non-trivial, for all finite α. Moreover, when collusion is sustainable only provided α takes appropriate values, it is worth stressing that the achievable degree of collusion remains constant, thus independent of n, along the path $(n, \alpha^*(n))$, where it takes its maximum value. Observe in fact that, along such path, the incentive compatibility constraint is met with equality for all n, i.e.

$$H(n, \alpha^*(n)) = \frac{1}{r}.$$

This is formally stated in the following proposition.

Proposition 2
Let $r \in (0,1)$. For all n such that $\bar{H}(n) > \frac{1}{r}$, the degree of collusion $H(n, \alpha)$ takes a maximum constant value strictly greater than one along the path $(n, \alpha^*(n))$.

Discussion of Proposition 2. It is worth comparing Proposition 2 with the existing literature on collusion in a Cournot setting. It is well known that, in the standard Cournot game, 'some' collusion can always be sustained regardless of the discount rate, or alternatively regardless of the number of firms for every $r > 0$. In other words, as the number of firms grows, there always exists a quantity less than the Cournot quantity, and a vector of profits that dominates the Cournot profits, on which collusion can be sustained. However, for any given $r > 0$, as the number of firms grows indefinitely, the 'achievable' collusive solution (in terms of both quantities and profits) steadily diverges from the 'full' cooperative solution (i.e. the market solution corresponding to the overall industry profits maximization when incentive constraints are ignored), and converges to the Cournot equilibrium. Therefore, in the standard Cournot game, the degree of collusion becomes trivial as the number of firms increases, for any given $r > 0$.

In our setting things are somehow different. For all n and α, $H(n, \alpha)$ equals the ratio of the profits in the 'full' cooperative solution to the Cournot profits.

Thus, provided that α is sufficiently large with respect to n, the achievable collusive solution always coincides, in our setting, with the solution that maximizes the overall industry profits. The point, however, is that the collusive solution itself, as well as the Cournot solution, depends on α. Therefore, we ask how the collusive solution behaves *vis à vis* the Cournot solution when α varies. On the one hand, one should observe that the collusive solution converges to the Cournot solution when α grows indefinitely, *given* n. This can be seen by considering that $H(n,\alpha) > 1$ for all α and for $n > 1$, $\partial H(n,\alpha)/\partial\alpha < 0$ for every $n > 1$, and $\lim_{\alpha\to+\infty} H(n,\alpha) = 1$. Hence, given n, the degree of collusion as measured by $H(n,\alpha)$ becomes trivial in the limit. On the other hand, however, provided that α takes the lowest value $(\alpha^*(n))$ that makes collusion sustainable for every n, Proposition 2 shows that the degree of collusion associated to all pairs $(n,\alpha^*(n))$ always keeps a constant, maximum, and non-trivial value greater than one, that depends on the discount rate but is independent of the number of firms.

It is worth noting that, although the degree of collusion $H(n,\alpha)$ keeps a constant, non-trivial value along the path $(n,\alpha^*(n))$, for the individual firm $\Pi_i^M(\alpha^*(n))$ – and a fortiori $\Pi_i^C(\alpha^*(n))$ – converges to zero along the same path. This can be seen by recalling that

$$\alpha^*(n) = b\left[\mu(n-1)-1\right], \tag{21}$$

where $\mu = \left((1-r)^{-1/2}-1\right)/2$. Thus, by substituting it in Equations (6) and (16), and rearranging, we obtain

$$\Pi_i^M(n,\alpha^*(n)) = \frac{a^2}{4b(\mu+1)(n-1)} \text{ and } \Pi_i^C(n,\alpha^*(n)) = \frac{a^2\mu}{b(2\mu+1)^2(n-1)}.$$

However, the aggregate collusive industry profits as well as the aggregate industry Cournot profits, i. e. $\Pi^M(n,\alpha^*(n)) = n\Pi_i^M(n,\alpha^*(n))$ and $\Pi^C(n,\alpha^*(n))$ $= n\Pi_i^C(n,\alpha^*(n))$ respectively, converge to a finite value bounded away from zero along the path $(n,\alpha^*(n))$, when n grows indefinitely. Moreover

$$\lim_{n\longrightarrow\infty}\left(\Pi^M(n,\alpha^*(n)) - \Pi^C(n,\alpha^*(n))\right) = \frac{a^2}{b}\cdot\frac{1}{4(\mu+1)(2\mu+1)} > 0.$$

Hence, in the aggregate, the collusive solution remains distinct from the Cournot solution whatever the number of firms.

As a consequence, along the same path $(n, \alpha^*(n))$, collusion implies a welfare loss never vanishing as the number of firms grows. In fact, the loss in consumer surplus induced by collusion (i.e. $CS^C(n, \alpha^*(n)) - CS^M(n, \alpha^*(n))$) converges to the constant and strictly positive value

$$\lim_{n \longrightarrow \infty} \left(CS^C(n, \alpha^*(n)) - CS^M(n, \alpha^*(n)) \right) = \frac{a^2}{8b} \cdot \frac{3 + 4\mu}{(\mu + 1)^2 (2\mu + 1)^2}. \quad (22)$$

Alternative measures of the degree of collusion. Proposition 1 measures the degree of collusion by the ratio of the collusive profits to the Cournot profits. Analogous results can be obtained by employing alternative measures of the degree of collusion: e.g., the ratio of the collusive quantity to the Cournot quantity, or the ratio of the collusive price to the Cournot equilibrium price.

Dividing Equation (4) by Equation (14) and Equation (5) by Equation (15) one obtains

$$\frac{q_i^M}{q_i^C}(\alpha, n) = \frac{2\alpha + b(n+1)}{2(\alpha + nb)} \text{ and } \frac{p^M}{p^C}(\alpha, n) = \frac{(2\alpha + nb)(2\alpha + nb + b)}{2(\alpha + nb)(2\alpha + b)}.$$

By calculating the ratios above along the path $(n, \alpha^*(n))$ we get that, independently of n:

$$\frac{q_i^M}{q_i^C}(n, \alpha^*(n)) = \frac{2\mu + 1}{2(\mu + 1)} < 1 \text{ and } \frac{p^M}{p^C}(n, \alpha^*(n)) = \frac{(2\mu + 1)^2}{4\mu^2 + 4\mu} > 1.$$

Furthermore, observe that $\partial \frac{q_i^M}{q_i^C}(\alpha, n) / \partial \alpha > 0$ and $\partial \frac{p_i^M}{p_i^C}(\alpha, n) / \partial \alpha < 0$. Hence, for all $\alpha \in [\alpha^*(n), +\infty)$, one has that $\frac{q_i^M}{q_i^C}(\alpha, n) > \frac{q_i^M}{q_i^C}(n, \alpha^*(n))$ and $\frac{p_i^M}{p_i^C}(\alpha, n) < \frac{p^M}{p^C}(n, \alpha^*(n))$, meaning that no greater degree of collusion can be achieved for all α and n such that $H(n, \alpha) \cdot r \leq 1$.

Harsher punishments. In our model the punishment phase is based on infinite reversion to the Cournot-Nash equilibrium. As the cost function exhibits decreasing returns, infinite reversion to the Cournot-Nash equilibrium implies that the present value of the stream of profits in the punishment phase in the incentive compatibility constraint, i.e. $V_i^C = \frac{\delta}{1-\delta}\Pi_i^C$, is strictly greater than zero, independently of n. It is well known that the scope for collusive behavior can be usually 'enlarged' by designing harsher, Abreu-type, punishment phases. The general result can be stated by saying that, under harsher punishment schemes, collusion can be sustained for higher values of the discount rate than those required under Cournot reversion (Abreu, 1986). In our setting such general result can be intuitively restated in the sense that, for any n and for any given discount rate, the minimum value of α which sustains collusion under harsher punishment phases is lower than $\alpha^*(n)$. To briefly illustrate the point, consider the extreme specific case in which there exists an 'optimal' penal code entailing $V_i^{PC} = 0$.[8] When $V_i^{PC} = 0$, the incentive compatibility constraint

$$\frac{1}{1-\delta}\Pi_i^M \geq \Pi_i^D + V_i^{PC} \tag{23}$$

becomes

$$r \leq \frac{\Pi_i^M}{\Pi_i^D - \Pi_i^M}. \tag{24}$$

Recalling that $H(n,\alpha) = \Pi_i^D/\Pi_i^M$, Inequality (24) can be written as

$$(H(n,\alpha) - 1) \cdot r \leq 1. \tag{25}$$

It is straightforward to see that Inequality (25) always holds for all pairs (n,α) for which Proposition 1 is true. Moreover, it is interesting to observe that, when $V_i^{PC} = 0$, there always exists a pair (n,α) such that (25) holds *for any value* of the discount rate r.[9] Finally, for all r such that $(H(n,0) - 1) > 1/r$, let $\breve{\alpha}(n)$ solve $(H(n,\breve{\alpha}(n)) - 1) \cdot r = 1$. Simple algebra shows that $\breve{\alpha}(n) = b[\breve{\mu}(n-1) - 1]$, where $\breve{\mu} = \frac{1}{2}\left(\sqrt{r+1} - 1\right)$. Since $\breve{\alpha}(n+1) - \breve{\alpha}(n) > 0$, $\breve{\alpha}(n)$ is increasing in n, a

[8] Observe that any punishment phase implying $V_i^{PC} < 0$ would indeed violate the individual rationality constraints of firms, as the latter always have the outside option of leaving the market in order to cut losses.

[9] This follows from $\lim_{\alpha\to+\infty} H(n,\alpha) = 1$ for $n > 1$.

result equivalent to the one obtained under Cournot-Nash reversion. Given that $H(n, \alpha)$ is increasing in n and decreasing in α, then for all r and n the incentive compatibility constraint always binds at $(H(n, \breve{\alpha}) - 1) \cdot r = 1$ for smaller values of α than those at which the incentive compatibility constraint under the Cournot-Nash reversion is binding (i.e. $H(n, \alpha^*) \cdot r = 1$).

3 Implications for Competition Policy

In this paper we show that there always exist technological conditions under which a collusive equilibrium among a large (potentially infinite) number of firms can be sustained for any given discount rate. Such technological conditions are linked with decreasing returns to scale. Moreover, we prove that the aggregate collusive profits in the industry converge to a finite value bounded away from zero, when the number of firms grows indefinitely, implying that the degree of collusion remains constant and non-trivial, and therefore distinct from the Cournot solution, whatever the number of firms. In our view, both results have important implications in the perspective of competition policy.

A first, straightforward, implication is that a Competition Authority should never ignore the likelihood of collusion simply relying on a "very crude and general rule" based on the large number of firms, as Scherer and Ross would suggest. More precisely, even within the structuralist approach, a careful investigation of the industry technology should be added to the analysis of industry concentration before any conclusion can be drawn on the likelihood of collusion. Reliable estimates of the parameters of the cost function would in fact help the Competition Authority to better evaluate, for plausible values of the discount factor, the level of industry concentration above which collusion becomes sustainable.

However, the implications for competition policy go further than what might be suggested by remaining within a – possibly more sophisticated – structuralist approach. Producers' may coordinate themselves not only through tacit or explicit collusion on strategic variables (such as prices and quantities), but also by devising institutional arrangements aimed at 'artificially' creating conditions that facilitate collusive behavior. Although so far little attention has been devoted to the institutional underpinnings of collusive behavior, institutional arrangements in specific industries are of particular relevance to understand how incentives to collude work, or may be manipulated. In industries where the efficient size of firms is 'small', the results obtained in Section 2 are especially useful to understand collusion in structurally dispersed sectors. Such results suggest in fact that, when the number of firms is too 'high' for the incentive compatibility constraint to be met (given r and α), firms may artificially 'distort' their marginal cost function by raising its slope in order to facilitate collusion.

In several economic systems we observe many industries organized according to internal rules that are reminiscent of traditional 'corporatism'. By and large, the concerned industries mainly pertain to the service sector and are structurally dispersed. In such industries, the relationships among a large number of firms are commonly governed by a number of rules explicitly or implicitly aimed at hindering the individual growth of the single producer at the expenses of its competitors. For instance, in many European countries, among which we include Italy, the provision of a vast range of professional services is ruled by deontological codes that condemn the 'unfair' subtraction of the competitors' customers. Whereas such deontological norms cannot extend to the point that they explicitly preclude competition in the market, still other, more sophisticated, institutional arrangements that pursue similar goals are often at work. Consistently with the analysis in Section 2, we frequently observe that such rules have the main effect of making costlier the individual firm's decision to grow competitively.

We give two examples, drawn from the Italian experience, that illustrate the point. In June 2006, a decree presented by the Italian economic development Minister, aimed at liberalizing a number of services sectors, introduced several measures concerning, among others, the cab sector. Taxi drivers fomented a political opposition to the overall decree. In particular, they resisted vigorously against a specific measure which intended to remove a previous regulation — according to which only taxi drivers owning and operating a single cab as individual entrepreneurs, but no firms owning several cab licenses together, were allowed to provide the service. Eventually, the status quo regulation was preserved.[10] In the light of the results shown in Section 2, this kept the taxi drivers' marginal production cost rapidly increasing at very low levels of the single firm's supply, with strong collusive effects.

The second example deals with rules disciplining compensation for services provided by professional lawyers. In Italy, all lawyers must be affiliated to the *Ordine degli Avvocati*, a public organization that governs several aspects of the lawyers' activity. In particular, the payments a lawyer requests for her services are typically parametrized on fees indicated by the *Ordine degli Avvocati*. In case a customer refuses to meet the payment, the congruence of the lawyer's request with the fees indicated by the *Ordine*, when stated by the *Ordine* itself, is legally a sufficient element for a court to endorse the lawyer's claim. This provision greatly reduces the risks for the lawyer and provides her with a valuable insurance.An obvious consequence is that the enlargement of the business activity pursued through a deviation from the (collusive) fees indicated by the *Ordine* implies loosing the support of the *Ordine*, and for this reason proves to be extremely costly to a lawyer.[11]

Needless to say, institutional arrangements, such as those described above, require coordination that cannot rely on self-enforcement. In the theoretical setting of Section 2, a firm, considering deviation from tacit or explicit collusion on quantities, would obviously include in the deviation strategy also the restoration of the 'original' cost function, thus making collusion unstable. However, the collusive equilibrium can be sustained if the artificial distortion of the slope of

[10] See the Economist, August 5th 2006, p.27.

[11] We are grateful to Daniela Marchesi for attracting our attention to this example.

the marginal cost function is itself the content of a formal agreement enforceable by courts (for instance, because it goes unchallenged under antitrust law, for lack of sophisticated analysis) or is even enforced by a public regulation, as in the two examples given above.

In a different perspective, the possibility of firms' agreements or other arrangements aimed at easing collusive behavior lies also at the root of the notion of 'facilitating practices' in antitrust law. Facilitating practices can be defined as mechanisms that firms artificially design to change the market environment in such a way as to relax the incentive constraint for every firm to collude. As anticipated in the introductory section, in antitrust case law the organization of an information exchange is explicitly interpreted as a practice that facilitates collusion through the 'control' of the reaction time to a deviation. We believe that our analysis may open the way for detecting yet a different class of facilitating practices that appear to be of a particular relevance in dispersed oligopolies.

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