Socially Optimal Procurement with Tight Budgets and Rationing

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April 11, 2006

Abstract

We investigate the behaviour of a Social Planner that secures the supply of a public good through contracting with private agents. The Social Planner maximizes social net benefits under asymmetric information about production costs, while controlled with a fixed budget and. We find that the tight budget changes the optimal contract design in several ways: the social planner minimizes costs, rations high-cost contracts to reduce fixed costs and information rents and distorts contracts for all agent types, including low-cost agents. Also that even though social costs are taken into consideration, they do not affect the optimal contracts, when budgets are tight - in that case the budget constraint dominates the solution.

Keywords: Principle-Agent, tax distortion, social cost, budget constraint, rationing

JEL: D45, D82, Q5, Q58

1 Introduction

1.1 Public procurement

The provision of public goods is often ensured by the society through public supply, e.g. defence, health care, cultural goods or environmental goods. However public ownership is often not feasible or does not secure an adequate supply. Instead society needs to secure a supply from private agents by creating incentives schemes for output of the good. An example is the subsidies that are paid to landowners who supply environmental goods. Another is compensations to owners of historical buildings and sites that maintain a certain standard. In the following we will primarily use examples from procurement of environmental goods, but the model that we present here is applicable to other fields within public procurement too.

When designing contracts for public procurement of privately produced goods, society seeks to compensate only the true opportunity cost of the private agent. However, very often the opportunity cost of the private agent is unknown to the society, and the well-known problem of asymmetric information arises. The agent holds private information about his own costs, which enables him to collect information rents. Since the seminal work of Akerlof (1970), ways to handle such asymmetric information problems have been investigated in several ways and are now textbook material (e.g.Salanié (2001)). In principal-agent models of public procurement the objective of the society is usually to maximize the difference between social benefits and social costs by offering incentive schemes, which reduces the cost of rent-seeking behavior. Therefore it is often stressed in the political debate to take social costs and benefits into account, e.g. through the use of cost-benefit analysis. The purpose is to secure society '(environmental) value for money', a demand made continuously in the latter years policy debate on environmental challenges and economics, most notably in the debate initiated by Lomborg (2001).

However principals are often *also* working under a limited budget. That is, they are asked to maximize net social benefits subject to given budget constraints. The motives for such budget limitations may be many. In fact, any organization with multiple administrative or political layers faces coordination and motivation problems and the use of fixed budget allocations for sub-units is a commonly used instrument.

There is a number of authors that have looked at mechanism design when the buyer faces a restricted budget. Gautier (2004) studies a procurement problem, where the principal is financially constrained. He shows that it leads to underprovision of public goods and in a two type case it can be optimal to either offer two contracts or to pool the types depending on the size of the budget and the difference between the types. If it is not possible to separate the two types the design is a third best solution. Gautier and Mitra (2006) deal with regulating a monopolist with unknown cost. Since they only want to contract with a single agent, they find that bunching of efficient agents and separating of less efficient types are optimal. They also show that under restricted budgets, the principal can only achieve third best quantities, strictly lower than second best. Laffont and Roberts (1996) analyze an auction where the bidders have a common budget constraint. They show that the bidders offers are reduced compared to the unconstrained, even for bidders with low valuation of the good. Che and Gale (2000) relax the assumption that the constraint is common knowledge. They show that it is optimal to use price discrimination and offer a menu of contracts. Thomas (2002) deals with buyers with a common budget constraint, but different valuation of the good, unknown for the seller. He shows that for buyers with a binding budget constraint it is optimal to pool the agents to avoid information rent. For buyers that are not constrained by the budget, it is optimal to distort the contract upwards to make the budget constrained bind to mitigate the effect of the information rent. Levaggi (1999) looks at a procurement model with a limited budget and a varying output. She finds that pooling or a postcontractual bargaining is optimal in states of nature with low output to ensure that the budget is used. However, none of the authors considers the role of rationing the participation, which turns out to be very important in the presence of budget constraints. Nor do they consider the effects of combined social cost considerations and budget constraints.

Typical adverse selection models involve i) rationing away the participation of high costs types, cf. e.g. Antle, Bogetoft, and Stark (1999) and Antle, Bogetoft, and Stark (2001) ii) reducing the output from high-cost agents below their first best levels, cf. e.g. Demski and Sappington (1984). With a variable budget, the optimal rationing of the participation involves only the extreme cases - a given type is offered a contract with probability 0 or 1. However, combining the objective of maximizing net social benefits with a budget constraint it is plausible that the social planner can find it optimal not to contract with all the agents of a given type. Rather, budget constraints will perhaps stimulate the social planner to use some sort of rationing to reduce the burden of fixed costs or information rents. Rationing can be in the form of offering only a limited number of contracts for a given type of agents, allowing only some of the them to enter into an agreement. In practice this can be done as a lottery or simply as 'first come, first served'. The option to ration will allow the social planner to set contract terms different from those needed to bring all agents of a type into play, terms which may focus on lowering the burden of fixed costs and informations rents.

1.2 Our study

In this paper we investigate the role of social costs and rationing for a budget-constrained principal. We analyze the optimal behavior of this principal (in the following called the social planner) that contract with private agents to produce a good. Her objective is to maximize net social benefits subject to an exogenous budget constraint. Furthermore, she faces two types of agents, which differ in and possess private information about their costs. There are multiple agents of each type. The social planner is allowed to ration the contracts she offers to agents of each type.

We find that for tight budgets the solution to the social planners decision problem varies across four budget intervals, I_1, I_2, I_3 and I_4. For low budget intervals, in regime I_1, the social planner offers only contracts to the low-cost agents and rations if necessary. Once all low-cost agents are participating the social planer can involve the high-cost agents. That, however, will give low-cost agents a chance to collect information rents. Thus, to involve high-cost agents implies an increase in the cost of contracting with the low-cost agents. Therefore, once all low-cost agents are participating, there is a budget interval I_2, where the social planner increases the output in the low-cost contract until the marginal benefit-cost ratio drops to a level, where it becomes optimal to involve high-cost agents, in spite of information rents. In regime I_3 the social planner contracts with all low-cost agents and rations the participation of high-cost agents until they are all offered a contract. Then the social planner increases the output of both contract types (regime I_4), which continues until the budget is no longer binding.

The analysis reveales several interesting results. First of all, we find that for tight budget levels (regime I_1, I_2 and I_3) the optimal contracts are unaffected by social costs all together and instead dictated by the budget constraint and the option to ration fixed costs and, in regime I_2 and I_3, information rents. Part of this remains true also in I_4, as tax distortion has no direct impact on the optimal contracts. However as the budget constraint becomes less tight, the solution moves towards the unconstrained solution, where the optimal output depends on tax distortion. Thus, if budget constraints are tight, which barres the social planner from reaching the social optimum, social costs has limited or no effect on contract design.

Another interesting feature is the fact that the option to ration to avoid information rent brings about an upward distortion of low-cost agents - relative to the first-best case. Low-cost agents are distorted upwards in the I_3 and part of the I_4 regime, but the distortion levels out and disappears at the end of the I_4-regime, where the budget no longer is binding. For the high-cost agents we find a downward distortion of their output in I_3 and I_4 as well as in the unconstrained case.

Finally, the option to ration improves net social benefits, as it allows the social planner to "even out" the burden of fixed costs and "close convexity gaps" in the output set.

The paper is organized as follows: In the next section we describe the model and derive some basic characteristics of it. Following that we present our main analysis of the optimal procurement scheme with a binding budget constraint; first without the option to ration and then including this option. Next the model with an unlimited budget is presented. We use numerical illustrations to illustrate key-points. We point out the different distortions of the contracts, the gains from rationing and the role of social cost in the contract design. In a brief conclusion attention is drawn to the main results, caveats and potentials for extensions.

2 Model

The setting involves a principal, the social planner, and n agents. The agents have private information about their cost and is offered contracts by the social planner to produce an output. The output can be verified at no cost ex post by the social planner. Hereby, the informational problem is only involving adverse selection. We have chosen a model with specific functional forms for costs and benefits. This allows us to display the full richness of the results and ease the interpretation on communication of the results.

2.1 Principal and agents

There are two types of agents, a low-cost agent L and a high-cost agent H. Production of output a costs $v(a)$ for agents of type L and $hv(a)$ for agents of type H. Here, $h > 1$ reflects the cost inefficiency of the high-cost agent. For simplicity, the cost function has a simple quadratic form, $v(a) = a^2$, and both types have reservation utility Q. The agents are risk neutral and profit maximizers.

The fractions of H- and L-type agents of the total population are p_H and $p_L = 1 - p_H$, respectively. This is common knowledge among the agents and the principal. Also, without loss of generality we normalize the population to $n = 1$.

The social planner is given a budget $B > 0$, and she has explicit incentives to maximize the total net social benefits of the procurement scheme without overspending the budget. The procurement is funded by the use of distortionary taxes that cause a welfare economic loss of β per \$.

2.2 Procurement strategy

The procurement scheme consists of contracts that specifies output a_i and corresponding payments s_i. Since there are only two types of agents, it suffices to work with two types of contracts $\{a_L; s_L\}$ and $\{a_H; s_H\}$.

To allow for possible rationing of the participation, the social planner decides how large a proportion, $\alpha_L \in [0, 1]$ and $\alpha_H \in [0, 1]$, of each type of agent is offered to participate. Only a fraction α_L of the agents that signal or reveal to be low-cost agents will get a contract $\{a_L; s_L\}$, and similar for the agents that signal to be high-cost agents. An agent that is not awarded a contract is asked to produce 0 and is paid 0.

2.3 Contract design problem

The Social Planners's problem of designing the procurement scheme under asymmetric information can now be formalized as the following program

$$
\begin{array}{lll}
\max_{\alpha,a,s} & \begin{array}{l}\alpha_L p_L \left(a_L - \left(a_L^2 + Q\right)(1+\beta)\right) \\ +\alpha_H p_H \left(a_H - \left(h a_H^2 + Q\right)(1+\beta)\right) - \beta \alpha_H p_L \left(s_L - \left(a_L^2 + Q\right)\right)\end{array} & OB \\
s.t. & s_L - a_L^2 \geq Q & IR_L \\
 & s_H - h a_H^2 \geq Q & IR_H \\
 & \alpha_L (s_L - a_L^2) + (1-\alpha_L) Q \geq \alpha_H (s_H - a_H^2) + (1-\alpha_H) Q & IC_{LH} \\
 & \alpha_H (s_H - h a_H^2) + (1-\alpha_H) Q \geq \alpha_L (s_L - h a_L^2) + (1-\alpha_L) Q & IC_{HL} \\
 & \alpha_L p_L s_L + \alpha_H p_H s_H \leq B & BB
\end{array}
\tag{1}
$$

Here the objective function OB express the (expected) net social benefits of procurement, since a fraction α_L of the proportion p_L of low costs agents is offered a contract with output a_L and similar for the high-cost agents. For simplicity, the social benefits are assumed to be linear in a. In many cases the true social benefit functions of public goods are unknown or uncertain. Therefore, social benefits are simply a linear function of the production of the public good, a_L and a_H respectively.

The social cost enters OB with two different weights. The two first terms of OB are production costs, multiplied by the factor $(1+\beta)$ to take costs of tax distortion into account. The last term of OB reflects, as shown in Lemma 1 belo, that the optimal solution involves payment of information rent to the low-cost agents. However, in welfare economic terms will information rent only represent a transfer and is therefore multiplied with β.

The first two constraints, IR_L and IR_H are the individual rationality or participation constraints: the agents have to earn at least their reservation profits Q.

The next two constraints IC_{LH} and IC_{HL} are the incentive compatibility constraints that use the revelation principle (Myerson 1979) to ensure that it is in the best interest of agents to reveal their true type. IC_{LH} ensures that it will not pay for a low-cost agent to imitate a high-cost agent and IC_{HL} ensures that a high-cost agent does not profit from claiming to be a low-cost agent.

The last constraint is the budget balancing constraint that states that the social planner cannot overspend her budget. The constraint is formulated in expected terms, i.e. the expected payment to the agents cannot exceed the budget. From a practical perspective this models the idea of a fixed budget but with the flexibility to even out the budget allowance over time via a capital market. Alternatively, if there is a sufficiently large number of agents, n, we can always choose a rationing principle that comes arbitrarily close to the BB constraint: The number of low-cost agents is np_L so we shall ideally hand out $np_L\alpha_L$ contracts to low agents. Instead we can make m_L contracts, where m_L is the largest integer below $np_L\alpha_L$, and use any queuing principle to hand them out among the applicants.

2.4 Basic characteristics

In the standard two type adverse selection model without rationing, i.e. the $\alpha_L = \alpha_H = 1$ case, the high-cost agent has a binding participation constraint and the low-cost agent a binding incentive compatibility constraint, cf. e.g. Demski and Sappington (1984). In this case, then, it follows directly that in an optimal solution:

$$\begin{aligned} s_H &= h a_H^2 + Q \\ s_L &= (a_L^2 + Q) + (h-1) a_H^2 \end{aligned}$$

In this solution, the high-cost agent earns no extra-ordinary profit. The low-cost agent, however, earns an information rent of $(h-1)a_H^2$ due to the gains he can achieve by imitating the high-cost agent.

In the general case with rationing, the solution is similar but not identical. We report this as a Lemma and provide a proof in appendix A.

Lemma 1 *In an optimal solution to the procurement design problem we have* $\alpha_L > 0$ *and*

$$\begin{aligned} s_H &= h a_H^2 + Q \ \textit{(if } \alpha_H > 0\textit{)} \\ s_L &= a_L^2 + Q + \frac{\alpha_H}{\alpha_L}(h-1) a_H^2 \end{aligned}$$

The intuition of this characterization is that the information rent to the low-cost agent depends not only on the contracts offered to the high-cost agent, but also on the probability of getting such a contract. The larger a_H, the larger the rents as in the standard case. Now, however, the temptation increases in the probability of getting such a contract and decreases in the probability of getting an L-contract.

This simplifies the problem, since we know how to choose payments s_L and s_H once output and contract probabilities are determined. What remains is therefore to consider output levels a_L and a_H and contract probabilities α_L and α_H. The binding incentive and participation constraints are inserted into (1) to give the final model with a binding budget:

$$\begin{aligned} \max_{a_L, a_H, \alpha_L, \alpha_H} W &= \alpha_L p_L \left(a_L - \left(a_L^2 + Q\right)(1+\beta)\right) \qquad (2) \\ &\quad + \alpha_H p_H \left(a_H - \left(a_H^2 + Q\right)(1+\beta)\right) - \alpha_H p_L (h-1) a_H^2 \\ st. \quad B &= \alpha_L p_L (a_L^2 + Q) + \alpha_H p_H (h a_H^2 + Q) + \alpha_H p_L (h-1) a_H^2 \end{aligned}$$

3 The social planner with a budget constraint

Now we turn to the solution of the social planners procurement problem. First we look at the case where the social planner is not allowed to ration and therefore has to choose a solution where either all or none of a certain type of agents is offered a contract. Next the solution is presented, where rationing is a possible tool. We uncover the solution in steps, each of these corresponding to a budget interval in which the solution is distinctly different from the solution at other budget levels. The social planner chooses output levels, contracts probabilities and payments, $(a_L, a_H, \alpha_L, \alpha_H, s_L, s_H)$, as a function of the budget B. We derive the solution gradually and each step contains useful intuition.

To get started, we distinguish between two different situations corresponding to no use or some use of the high costs agents, $\alpha_H = 0$ or $\alpha_H > 0$.

Only the case of asymmetric information is presented in detail. The first-best solution is only mentioned when it highlighted the second-best solution. The full first-best model is given in Appendix C.

3.1 All or none solutions

Consider first the non-rationing solution where all or none of a given type is offered a contract. There are four possible situations $(\alpha_L, \alpha_H) \in \{0,1\} \times \{0,1\}$ but not all combinations are relevant. In fact, only the two possibilities $(\alpha_L, \alpha_H) = (1,0)$, $(\alpha_L, \alpha_H) = (1,1)$ can be optimal when $B > 0$. We record the non-rationing solution in Lemma 2 below for the two cases.

Lemma 2 *When only none or all agents of a given type are offered a contract, the social planner will select the set of contracts, which provides the largest net social benefit and satisfies the following in each of the two cases:*

When $(\alpha_L, \alpha_H) = (1,0)$ the optimal solution is given by:

$$\begin{array}{ll} \alpha_L = 1 & \alpha_H = 0 \\ a_L = \sqrt{\frac{B}{p_L} - Q} & a_H = 0 \\ s_L = a_L^2 + Q & s_H = 0 \end{array}$$

When $(\alpha_L, \alpha_H) = (1,1)$ the optimal solution satisfies:

$$\begin{array}{ll} \alpha_L = 1 & \alpha_H = 1 \\ s_L = a_L^2 + Q + (h-1)a_H^2 & s_H = h a_H^2 + Q \end{array}$$

and the optimal output levels a_L and a_H requested satisfies:

$$a_L = \frac{a_H(h + \widehat{h})}{(1 + 2\widehat{h} a_H)}$$

and

$$\frac{(B-Q)}{p_H} = \frac{p_L}{p_H} \left(\frac{a_H^2 (h + \widehat{h})^2}{(1 + 2\widehat{h} a_H)^2} \right) + \left(h + \hat{h} \right) a_H^2$$

where $\hat{h} = \frac{p_L}{p_H}(h-1)$ and a_H is the single non-negative root of the polynomium, implied by the last relation.

The proof of this Lemma is given in Appendix A. It is optimal to use only the low-cost agents for small budgets and to use both types of agents for higher budget values. The budget level where the shift occurres depends on the parameters of the problem. In Figure 1 we map the social cost and social benefits curves for the social planner when she is not allowed to ration.

[Insert figure 1 somewhere here. Caption text: The social benefits and social cost curves, when the principal is not allowed to ration ($p_L = 0.5, p_H = 0.5, h = 2, Q = 0.05, \beta = 0.2$).]

Without rationing, the social planner needs to offer all or no agents of a given type a contract. This implies a heavy burden of fixed costs as illustrated in Figure 1, where the social planner has to pay fixed cost of $p_L Q$ before she get any benefits. One of the advantages of rationing is that it allows the principal to involve fewer agents and hence save fixed costs, as we shall see below. Put differently, *rationing enables the principal to take advantage of economies of scale for small budget values.*

Without rationing the minimal cost curve is non-convex for medium cost levels. An additional advantage that we demonstrate below as well, is that probabilistic *rationing enables the principal to even out these non-convexities for medium budget values* and hereby to increase the procurement.

3.2 Including only low-cost agents

When $\alpha_H = 0$ the social planner chooses to involve only the low-cost agents. The social planner knows the proportion p_L of low-cost agents and knows their cost function. Therefore, for low budgets, the social planner can contract only with the low-cost agent. The contract design problem when $\alpha_H = 0$ is - using Lemma 1 - reduced to

$$\begin{aligned} \max \quad & \alpha_L p_L \left[a_L - (1+\beta)\left(a_L^2 + Q\right)\right] \\ s.t. \quad & B \geq \alpha_L p_L \left(a_L^2 + Q\right) \end{aligned}$$

With only one contract designed for the low-cost agent, there is no problem with asymmetric information, since the high-cost agent has a negative profit of accepting this contract and he therefore never participates.

3.2.1 $I1$: Rationing the low-cost agents

When the budget is very low, the principal rations contracts to the low-cost agents. The solution is for $\alpha_L < 1$:

$$a_L = \sqrt{Q} \tag{3}$$

$$\alpha_L = \frac{B}{2Qp_L} \tag{4}$$

The intuition of this solution is that the social planner first chooses the output level to make the average costs as small as possible, i.e. she minimizes

$(a_L^2 + Q)/a_L$. Next she determines how many contracts to offer to spend the budget, i.e. she chooses α_L to get $\alpha_L p_L(a_L^2 + Q) = B$. This makes the principal able to decrease the fixed cost paid per contract Q and is necessary to minimize average costs. All low-cost agents are alike, therefore they are offered the same contract$\{\sqrt{Q}; 2Q\}$. The first-best solution is identical since there is no informational problem.

3.2.2 $I2$: Increasing output and payment

Once all low-cost agents are offered a contract, the social planner can choose to introduce the high-cost agents, but then she has to pay information rents to all the agents of the low-cost type, cf. Lemma 1. Therefore it is optimal to increase the output required from the low-cost agents. This solution is identical to the case where the principal is not allowed to ration (Lemma 2):

$$a_L = \sqrt{\frac{B}{p_L} - Q}. \tag{5}$$

The social planner increases the output required from the low-cost agents before involving the high-cost agents, *only* as long as the benefit-cost ratio is larger for the former option. In this interval I_2 the social planner chooses to distort the low-cost agent above their minimum average cost solution, depending on the magnitude of the information rents upon involvement of the high-cost agent and the option to ration against them.

The intuition of this solution is that since the number of participants will not increase, the low-cost agents are instead offered contracts with higher output and payments. The agents operate beyond the most productive scale size and experience some diseconomies of scale.

Since only one type is involved information rents are still absent and the profit of the agents is zero. Note, that the social cost related to tax distortion has no implication for the design of the contracts when budgets are tight.

3.3 Using both type of agents

When the high-cost agents are used at least on occasion, the low-cost agent will *always* be used, $\alpha_L = 1$. This follows from the fact that it is cheaper to use a low-cost than a high-cost agent at a given output level.

With $\alpha_L = 1$, the Social planner's full problem - using Lemma 1 - is

$$\begin{aligned}
\max\ W &= p_L\left[a_L - (1+\beta)\left(a_L^2 + Q\right)\right] \\
&\quad + \alpha_H p_H\left[a_H - (1+\beta)\left(h a_H^2 + Q\right)\right] - \alpha_H p_L \beta\,(h-1)\,a_H^2 \\
s.t.\ B &\geq p_L\left(a_L^2 + Q\right) + \alpha_H p_H\left(h a_H^2 + Q\right) + \alpha_H p_L\,(h-1)\,a_H^2.
\end{aligned}$$

3.3.1 $I3$: Rationing the high-cost agents

The solution is now somewhat more complicated when $\alpha_H \in (0,1)$ (cf. Appendix B for more details):

$$
\begin{aligned}
a_L &= \frac{\sqrt{Q}(\widehat{h}+h)}{\widehat{h}\sqrt{Q}+\sqrt{\widehat{h}^2 Q+\widehat{h}+h}} && (6) \\
a_H &= \frac{\widehat{h}Q+\sqrt{(\widehat{h}Q)^2+(\widehat{h}+h)Q}}{(\widehat{h}+h)} && (7) \\
\alpha_H &= \frac{B-p_L\left(a_L^2+Q\right)}{p_H Q+p_H(\widehat{h}+h)a_H^2} && (8)
\end{aligned}
$$

We use $\widehat{h}=(h-1)\frac{p_L}{p_H}$ as in Lemma 2. Note, that over this interval the outputs a_L and a_H are constant. We also note from the solution that $Q=a_H a_L$, i.e. fixed cost minimization is still a driving factor. This relation is also found in the first-best case, where $a_L=\sqrt{Qh}$ and $a_H=\sqrt{Q/h}$ (see appendix C). The solution here seems much more complicated, but the only difference is in fact the cost of information rents represented by $\hat{h}$. To verify that simply set $\hat{h}=0$ in the second-best solution of (6) and (7), and see that it reduces to the first-best solution.

We see, that the additional costs related to information rents - and the option to ration against them - cause the social planner to distort *both* types symmetrically. The proof is given in Appendix B. There we show that $a_L^{SB}>a_L^{FB}$ for all contracts with a positive net social benefit, which means that there is always an upwards distortion of the output when there is asymmetric information. From this follows that the second-best solution for the high-cost contract in regime I_3 is distorted downwards compared to the first-best, since it in both cases holds that $Q=a_L * a_H$. This ensures the minimization of average cost and distributes the burden of information rent equally on both agent types. The solutions is still dictated by the budget constraint and social costs of tax distortion remain irrelevant for the design of the contracts.

To summarize the solution, the social planner chooses to ration against high-cost agents by lowering their output and by offering contracts with probability less than one. The first saves information rents to the low-cost agents but comes at the cost of having more fixed costs. The second approach also saves information rents and it comes with the added advantage of reducing the fixed costs by calling upon less high-cost agents to produce. The social planner also distorts the low-cost agents upwards. This is attractive to spend the budget without having to use too many high-cost agents (with their direct costs of output and their indirect costs of becoming attractive to imitate).

3.3.2 *I*4: Increasing procurement towards the unconstrained model

Above B_3, all agents of both types are offered a contract. For larger budgets, the social planner chooses to increase the output of the contracts, but will do so only as long as the budget still binds. It is implicit that the budget is binding over regime I_3. Therefore the social planner will enter into this fourth regime I_4 where $\alpha_L = \alpha_H = 1$. This is identical with the problem in Lemma 2, when the principal is offering two contracts. We find a relation between a_L and a_H identical to that found in regime I_3:

$$a_L = \frac{a_H(p_H h + p_L(h-1))}{p_H + 2p_L(h-1)a_H} = \frac{a_H(h+\widehat{h})}{(1+2\widehat{h}a_H)}. \tag{9}$$

And that the high-cost contract shall fulfil the following expression:

$$\frac{(B-Q)}{p_H} = \frac{p_L}{p_H}\left(\frac{a_H^2(h+\widehat{h})^2}{(1+2\widehat{h}a_H)^2}\right) + \left(h+\hat{h}\right)a_H^2 \tag{10}$$

which is a fourth-order polynomium, where a_H is given by the single positive root. One can show (after an excessive amount of tedious algebra), that the solution to the unconstrained social planner's problem, see (15), is a solution to this equation and so is the solution from regime I_3, see (6-6). The latter forms the starting point of regime I_4 and the former implies the ending point of regime I_4, where the budget is no longer binding. This equals the solution where the unconstrained social planners is offering contracts to both types of agent (see section X).

3.4 The solution

Now four different regimes have been identified. What remains is to determine in which budget intervals each regime is relevant and whether they are unique solutions.

The budget limit for I_1 can be determined as the budget, where all the low-cost agents are offered the contract specified in (3) which is the same as when $\alpha_L = 1$. We call this B_1:

$$B_1 = 2p_L Q \tag{11}$$

The budget level, B_2, where the social planner will start to involve the high-cost agent can be found from (8) by setting $\alpha_H = 0$:

$$B_2 = p_L\left(a_L^2 + Q\right), \tag{12}$$

where a_L is given by (6) in the second-best case and (25) in the case of symmetric information. When $\alpha_H = 1$ in the same equation, we have $B = B_3$:

$$B_3 = B_2 + p_H\left(\left(\widehat{h}+h\right)a_H^2 + Q\right), \tag{13}$$

and a_H is given by (7) in the second-best case and (26) in the case of symmetric information.

Now we can summarize the solution in a Proposition

Proposition 1 *The optimal solution to the social planner's problem is given by (3-4) for $B \leq B_1$, (5) for $B_1 \leq B \leq B_2$, (6-8) for $B_2 \leq B \leq B_3$ and (9-10) for $B_3 \leq B \leq B_4$. Moreover, $B_1 < B_2 < B_3 < B_4$ such that all regimes may be relevant.*

The Proposition follows quite directly from the analysis above and the final aspects, namely that the individual regimes will be patched together as in the Proposition, are proved in appendix A.

4 The unconstrained social planner

Before turning to the discussion we look at the model with an unlimited budget. This unconstrained social planner has the same objective as the constrained planner. However she is not binded by the budget and can therefore always reach the social optimal solution. Therefore the social planner's problem reduces to

$$\begin{aligned} W &= \alpha_L p_L \left(a_L - \left(a_L^2 + Q\right)(1+\beta)\right) \\ &\quad + \alpha_H p_H \left(a_H - \left(a_H^2 + Q\right)(1+\beta)\right) - \alpha_H p_L (h-1) a_H^2 \end{aligned}$$

Since the objective is linear in α_L and α_H, the optimal solution will never use randomized rationing. We record this as a Lemma.

Lemma 3 *The social planner does not use randomized rationing, i.e. she either contracts with all or none of a given type of agent, $\alpha_L \in \{0,1\}, \alpha_H \in \{0,1\}$.*

The intuition is that if it pays to use an agent type sometimes, it pays to use the agent all the time. For the low-cost agent, this is not surprising, but the high costs agent has the added disadvantage of creating information rents and one could therefore expect that it might pay to reduce the rents by lowering the probability of awarding high-cost agents a contract. This does not happen however, since the benefits in terms of additional output outweighs the extra production cost and information rents for all values of α_H if it does for just one value.

And the solution is

$$a_L = \frac{1}{2*(1+\beta)} \tag{14}$$

$$a_H = \frac{1}{2*\left(h*(1+\beta)+\beta*(h-1)\frac{p_L}{p_H}\right)}. \tag{15}$$

As the reader can easily verify himself, this solution differs from the first-best in the standard way. That is, the low-cost agent agents are offered to produce the same amount of goods as in the first-best case and get an information rent. For the high-cost agent agents, the solution differs from the first best since the last term in the denominator has been added. It reflects the social cost of information rents paid to the low-cost agents and since it is positive it implies the usual downward distortion in the output. The social planner offers these contracts to all agents of a given type, provided that the social benefit of the type is positive. Note that here in the unconstrained case, the cost related to tax distortion affects the optimal contract as it reduces the output level.

The budgetary cost of realizing the two-contract solution is B_4:

$$B_4 = Q + \frac{p_L}{4\left(1+\beta\right)^2} + \frac{p_H\left(h+\widehat{h}\right)}{4\left(h\left(1+\beta\right)+\beta\widehat{h}\right)^2} \tag{16}$$

This is the budget where the constrained social planner is no longer limited by her budget and therefore she does not utilize any budget above B_4 as mentioned in Proposition 3.

5 Discussion

5.1 Welfare gains from rationing

In most studies in the literature of principals with a limited budget, rationing is assumed not to take place (e.g. Levaggi (1999), Gautier and Mitra (2006), Gautier (2004)) and optimal contracts are determined subject to that fact. In our model the budget-constrained social planner picks the procurement scheme, which places her as far to the right as possible on the curves shown in Figure 1 for a given budget. The social planner does not offer any contracts for budgets $B \leq p_L * Q$ as she can at most cover fixed costs unless she is allowed to ration. With $B > p_L * Q$ the output of the low-cost contract will be increased, but slowly since she has to offer all low-cost agents a contract. Therefore we find some clear non-convexities in the minimum social cost curve of the optimal path for the social planner - seen as a function of available budget.

[Insert figure 2 somewhere here. Caption text. The effect of rationing when $\alpha_L < 1$. ($p_L = 0.5, p_H = 0.5, h = 2, Q = 0.05, \beta = 0.2$).]

Interestingly, the role of rationing is to 'bridge' these gaps to increase the welfare gain. In Figure 2 we see how rationing implies that the social planner can enter into procurement for much lower budgets (and hence social costs) than in the case of no rationing. In I_2, where all low-cost agents are in play, the minimum social cost curve coincides with the cost of using only low-cost agents.

[Insert figure 3 somewhere here. Caption text: The effect of rationing when both agents are involved and $\alpha_H < 1$. ($p_L = 0.5, p_H = 0.5, h = 2, Q = 0.05, \beta = 0.2$).]

In I_3 the principal chooses to ration the high-cost agents for two reasons: to save fixed cost - as in regime I_1 and to minimize the information rent paid to the low-cost agents. Again, rationing allowes the social planner to place herself - and society - on a lower minimum social cost curve, cf. Figure 3, which results in a welfare gain to society.

5.2 The effect of asymmetric information on contracts

We have looked at the second-best case as the primary case of interest and only mentioned the first-best solution briefly. In this section the differences between these cases is analyzed to reveal the effect of asymmetric information. We look at the optimal output level of the two types of agents and also compare the solutions to the unconstrained counterparts.

The optimal output levels a_L and a_H, depending on the budget, is seen in Figure 4. For low budget the first-best and the second-best solution are equal until the point where all the low-cost agents are involved. Then the output from them increases gradually as the budget increases at the same rate for both symmetric and asymmetric information regimes. Finally, it becomes optimal to involve the high-cost agents, and until they are all in play, the contracts of both agents are kept constant at the weighted average cost-minimizing level. Interestingly, the optimal contracts of the low–cost agents are distorted upwards in regime I_3 under asymmetric information to ensure cost-minimization. The low-cost agents are offered contracts involving higher levels of output - an unusual finding caused by the option to ration under budget constraints. Once all agents are in play - in the I_4 regime - the output levels are increased towards the unconstrained optimum (where the lines end). Outputs are slowly converging towards the unconstrained optimum, where the upwards distortion is no longer present. Instead the cost of paying information rents force the social planner to work with lower contracts for given budget levels. In both regime I_3 and I_4, the high-cost agents are offered contracts that are distorted downwards as in the standard literature.

Insert figure 4 somewhere here. Caption text: Optimal output levels depending on the available budget, information and agent type ($p_L = 0.5, p_H = 0.5, h = 2, Q = 0.05, \beta = 0.2$).]

Another effect of the asymmetric information is that the budget, where the optimal solution changes, is higher in second-best case. In Figure 4 it is easy to see that the $B_2^{SB} > B_2^{FB}$ and $B_3^{SB} > B_3^{FB}$ due to the extra cost of information rent under asymmetric information. In the second half of Appendix B we show that $B_3^{SB} - B_2^{SB} > B_3^{FB} - B_2^{FB}$; the regime continues longer in the second-best model, which means that the budget has to be larger before all agents are included in the scheme compared to the first-best situation. This is natural since offering a contract to an additional high-cost agent is more costly due to the increase in information rent.

5.3 The role of social costs with tight budgets

The optimal contract in the unconstrained case of the social planner's problem is (14) and (15). The marginal social cost to increase the output of the individual contract enters both equations. This includes the social cost of tax-distortion related to not only the output costs, but also the information rent transfer. Thus, social costs obviously matter directly to the unconstrained social planner and decrease the optimal output. This changes somewhat when budgets are tight.

When the budget is very tight in regime I_1, I_2 and I_3, the solution is independent of the social costs all together. For higher budget levels in regime I_4 the budget constraint still dominates the solution, and tax distortion has no direct influence on the design of the optimal contracts. The solution in I_4, however, differs from the case where social costs are ignored in the objective function of the principal: the social planner's solution converges towards the unconstrained solution along the minimum cost path. This is, as we have already pointed out, influenced by the social costs - also of tax distortion - and therefore the solution slowly becomes sensitive to the social costs.

An implication of these observation is, that when public agencies are required to optimize public procurement schemes in terms of net social benefits, and at the same time is kept in a very short leash budget-wise, they choose a contract design, which largely ignores the social costs. Note that this does not imply non-optimallity: Just that the very tight budget renders the call for social cost-benefit analysis superfluous.

6 Concluding Remarks

We find that for low budget levels the optimal contracts are unaffected by the social cost altogether, and instead entirely dictated by the budget constraint and the option to ration fixed costs and, in regime I_2 and I_3, information rents. Parts of this remains true also in the budget-constrained regime I_4, where the solution gradually converges towards the unconstrained solution where tax distortion does have an impact on the optimal contracts. It hinders the social planner from reaching the social optimum and therefore social costs have no or limited effect on the contract design. Instead the social planner displays a budget-cost minimizing behavior in her choice of contract. To draw a parallel from this point into a recent debate, consider the discussion following The Copenhagen Consensus Conference (http://www.copenhagenconsensus.com), where Bjørn Lomborg asked a group of distinguished economists to rank a number of efforts to address global issues related to hunger, poverty, wars and environmental degradation - all within a $50 billion budget. The setup and results had been subject to much debate, but here we will draw attention to a point made by Sachs (2004). Sachs points out, that given the scale of the problems, the budget - even if a very large number for any ordinary household budget - is in fact very tight (almost a factor 10 smaller than e.g. the US defence budget). Therefore, he argues, it is no surprise that the panel ends up focusing much on direct costs - in particular when much uncertainty on the benefits of various policy measures remained high during the conference. Our results provides some support for the claim, that even a social planner will be minimizing average cost if budgets are tight - and it is socially optimal to do so.

An interesting feature in this paper is the fact that the option to ration against information rents brings about an upward distortion of low-cost agents relative to the first-best case of symmetric information. Low-cost agents are distorted upwards in the I_3 regime, but with the distortion leveling out and disappearing over the I_4-regime. For the high-cost agents, we find a down-ward distortion of their output levels over the regimes I_3 and I_4 - in the constrained as well as in the unconstrained case. In the standard literature the 'No distortions at the top'-rule prevails (Salanié (2001)). However, a positive distortion of production levels does exist in other settings. If the agents have countervailing incentives, a positive distortion can be optimal for to decrease information rent for agents, that have incentive to understate their type (Maggi and Rodriguez-Clare (1995)). This is not distortion-at-the-top, but instead overproduction of some intermediate types. Lockwood (2000) shows that positive production externalities also makes overproduction optimal for agents with low marginal costs and underproduction for agents with high marginal costs. The last example comes from a model where agents have endogenous information about costs and information gathering is costly. In that case a positive distortion of production is optimal when the principal wants to encourage information-gathering activities (and the first-order approach does not hold) (Szalay (2005)). However, we know of no examples, where it is a constrained budget, that trickers the overproduction for the most efficient agents.

Finally, we see that the option to ration improves net social benefits, as it allows the social planner to "even out" the burden of fixed costs and "close convexity gaps" in the output set. This is impossible without rationing, but of course we should note, that the convexities exist because of the significant fixed costs in our model and our choice of a simple distribution of two discretely different agent types.

The setup with only two agent types used in this paper has been fairly simple and we have chosen explicit and straightforward functional forms to describe their differences as well as the principal. Obvious extensions would be to allow for more general distributions of agent types, where we expect much the same results to hold true, except that non-convexities will perhaps be less pronounced. Another issue to investigate would be the potential impact of transaction costs (see e.g. Crocker and Masten (1996)).

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7 Appendix A

In this appendix we present the proofs of Lemma 1, Lemma 2 and Proposition 3.

Proof. Proof of Lemma 1 ■

We first observe in the usual way that at least one of the IR-constraints has to be binding. This followed by way of a contradiction.

In the situation where we have an optimal solution (a_L, s_L, α_L) and (a_H, s_H, α_H) with none of the IR-constraints binding, we can fix the payments and increase the production levels slightly (and in the right proportions) without violating the IC constraints. This will be clear if we reformulate the IC constraints as

$$\begin{array}{ll} \alpha_L(s_L - a_L^2 - Q) \geq \alpha_H(s_H - a_H^2 - Q) & IC_{LH} \\ \alpha_H(s_H - ha_H^2 - Q) \geq \alpha_L(s_L - ha_L^2 - Q) & IC_{HL} \end{array}$$

If we choose the new production levels as $\bar{a}_L$ and $\bar{a}_H$ such that $\alpha_L(a_L^2 - \bar{a}_L^2) = \alpha_H(a_H^2 - \bar{a}_H^2)$ the IC constraints are not affected, and if we make $\bar{a}_L^2 - a_L^2 > 0$ and $\bar{a}_H^2 - a_H^2 > 0$ sufficiently small, the IR constraints will still not bind. The budget constraint will also remain non-violated, so the solution is still valid. The objective function will however improve and thereby contradicts the optimality of the solution in the first place. Now, this will not affect the budget constraints, so the feasibility of the solution will still be valid. The solution will however improve by the increased production.

Assume now that IR_L binds but that IR_H do not. In that case, $s_L - ha_L^2 + Q < 0$,and we can increase a_H marginally without changing s_H. This will improve the objective and we have a contradiction.

We therefore have proved that IR_H is binding in optimum, i.e.

$$s_H = ha_H^2 + Q$$

and what remains is only to characterize s_L. There are two possibilities. One is that $\alpha_H = 0$ such that the IC_{LH} constraint is basically redundant and we have from IR_L that $s_L = a_L^2 + Q$. If $\alpha_H > 0$, the right hand side of IC_{LH} is strictly positive and therefore IR_L will not bind. It follows that IC_{LH} has to bind since otherwise we can marginally increase a_L. From IC_{LH} binding we get

$$S_L = a_L^2 + Q + \frac{\alpha_H}{\alpha_L}(s_H - a_H^2 - Q)$$

and if we use that IR_H bind this can be rewritten as

$$S_L = a_L^2 + Q + \frac{\alpha_H}{\alpha_L}(h-1)a_H^2$$

QED.

Proof. Proof of Lemma 2 ■

In the case where only none or all agents of a given type are offered a contract, the social planner selects the set of contracts, which provides the largest net social benefit and satisfies the following in each of the two cases:

When $(\alpha_L, \alpha_H) = (1, 0)$ the optimal solution is identical to the solution in I_2.

When $(\alpha_L, \alpha_H) = (1, 1)$, we use the structure of the payments from Lemma 1. If they are inserted into the welfare function and the budget function we can determine the optimal contracts by solving:

$$\begin{aligned} & \max\ W = p_L \left[a_L - (1+\beta)\left(a_L^2 + Q\right)\right] \\ & +p_H \left[a_H - (1+\beta)\left(ha_H^2 + Q\right)\right] - p_L\beta (h-1) a_H^2 \\ s.t.\ \ B \quad \geq \quad & p_L \left(a_L^2 + Q\right) + p_H \left(ha_H^2 + Q\right) + p_L (h-1) a_H^2. \end{aligned}$$

The first order conditions are:

$$\begin{aligned} \frac{\delta L}{\delta a_L} &= p_L(1 - 2(1+\beta) a_L) - \lambda(2p_L a_L) = 0 \\ \frac{\delta L}{\delta a_H} &= p_H (1 - 2(1+\beta) h a_H) - 2p_L\beta (h-1) a_H \\ & \quad -\lambda (2p_H h a_H + 2p_L (h-1) a_H) = 0 \\ \frac{\delta L}{\delta \lambda} &= p_L \left(a_L^2 + Q\right) + p_H \left(ha_H^2 + Q\right) + p_L (h-1) a_H^2 - B = 0 \end{aligned}$$

and the optimal output levels a_L and a_H satisfied:

$$a_L = \frac{a_H(h + \widehat{h})}{(1 + 2\widehat{h} a_H)}$$

and

$$H(a_H) = \frac{(B-Q)}{p_H} - \frac{p_L}{p_H}\left(\frac{a_H^2 (h+\widehat{h})^2}{(1+2\widehat{h} a_H)^2}\right) - \left(h + \hat{h}\right) a_H^2 = 0 \qquad (17)$$

where $\hat{h} = \frac{p_L}{p_H}(h-1)$.

In the fourth-order polynomium we notice the following facts directly from the expression (17) and that all parameters are non-negative by definition. Firstly, since the fourth-order terms had a negative sign, we have a polynomium, with 'downward' bending legs. Secondly, for $2\widehat{h} a_H = -1$, the function $H(a_H)$ is not defined (the value is "infinitely negative"). Thirdly, $H(a_H)$ has a local maximum for $a_H = 0$ with the value $(B-Q)/p_H$. Being a fourth-order polynomium, there is also a local maximum for some $a_H < -1/2\widehat{h}$; this maximum is of course irrelevant for our problem. Combining these facts it logically followed that $H\prime(a_H) < 0$ for $a_H > 0$. Thus, there is at most one non-negative root of this polynomium, and in fact for $B > Q$ we always have exactly one positive root. Therefore, if - using numerical methods - one can identify a positive root of $H(a_H)$, that root is known to be the optimal level of a_H. Then the optimal level of a_L follows from the relationship between these identified in (**??**).

QED.

Proof. Proof of Proposition 3 ∎

The characteristics of the individual regimes are derived in the main text. What remained is basically to prove that they can be patched together. It is easy to see that $B_2 < B_3$ in both the first-best solution and the second-best solution. Also $B_1 > B_2$ in the first-best case. In appendix B we show that this is also the case in the second-best case. Therefore we know that

$$B_1 = 2p_L Q < B_2 = p_L(a_L^2 + Q) < B_3 = B_2 + p_H\left(\left(h + \widehat{h}\right) a_H^2 + Q\right)$$

for the second-best case, since $a_L^2 > Q$ in interval I_2 and $\widehat{h} = (h-1)\frac{p_L}{p_H} > 0$.

In the analysis of the four regimes, we showed that if we use only one type, we have to choose solution (3) for $B \leq B_1$, and (5) for $B_1 \leq B$. Likewise, we showed that when we use two types, we have to use (6-7) for $B_2 \leq B \leq B_3$ and (9-10) for $B_3 \leq B$.What remains is therefore to show that (5) can not be attractive for $B > B_2$ and that no two-type regime can be optimal for $B < B_2$. However, this is simple.

It cannot be attractive to use (5) for $B > B_2$ since $(\alpha_L, \alpha_H) = (1, 0)$ is a special case of the situation we analyzed with two agents, i.e. a possible solution to problem **??**, and in the analysis we found that (6) has to be used for any B in (B_2, B_3).

The possibility of using a two type regime for $B \leq B_2$ is also not optimal. There are two possibilities. One is to have $\alpha_L < 1$, but this cannot be optimal together with $\alpha_H > 0$. It will be better to use a larger budget share on the low-cost agents since they have lower costs and do not give other agents access to information rents. The other possibility is to have $\alpha_L = 1$ and $\alpha_H > 0$ but then we are back to the situation analyzed in **??**. An inner solution here required B in (B_2, B_3). Therefore to be optimal for $B < B_2$, it has to be a boundary solution with $\alpha_H = 0$ in contradiction with a two-type solution.

8 Appendix B

8.1 Second-best regime I_3: Details of the solution

In this appendix B, we present details of the solution in regime I_3.The problem that the principal faces in this regime is:

$$\begin{aligned}
\max\ W &= p_L\left[a_L - (1+\beta)\left(a_L^2 + Q\right)\right] \\
&\quad + \alpha_H p_H\left[a_H - (1+\beta)\left(h a_H^2 + Q\right)\right] - \alpha_H p_L \beta\,(h-1)\,a_H^2 \\
s.t.\ \ B &\geq p_L\left(a_L^2 + Q\right) + \alpha_H p_H\left(h a_H^2 + Q\right) + \alpha_H p_L\,(h-1)\,a_H^2.
\end{aligned}$$

The first order conditions can be derived from the Lagrangian as:

$$\frac{\partial L}{\partial a_L} = p_L \left[1 - (1+\beta)\, 2a_L\right] - \lambda 2 p_L a_L = 0 \tag{18}$$

$$\frac{\partial L}{\partial a_H} = \alpha_H p_H \left[1 - (1+\beta)\, 2ha_H\right] - \alpha_H p_L \beta\, (h-1)\, 2a_H \tag{19}$$
$$-\lambda \left[\alpha_H p_H 2ha_H + \alpha_H p_L\, (h-1)\, 2a_H\right] = 0$$

$$\frac{\partial L}{\partial \alpha_H} = p_H \left[a_H - (1+\beta)\left(ha_H^2 + Q\right)\right] - p_L \beta\, (h-1)\, a_H^2 \tag{20}$$
$$-\lambda \left[p_H \left(ha_H^2 + Q\right) + p_L\, (h-1)\, a_H^2\right] = 0$$

We substitute (18) into (19) to get the following relation between a_L and a_H:

$$a_H \left(h + \widehat{h}\right) = a_L \left(1 + 2a_H \widehat{h}\right),$$

where $\widehat{h} = (h-1)\frac{p_L}{p_H}$. Then we substitute (19) into (20) and get:

$$\frac{a_H^2 * \left(h + \widehat{h}\right)}{\left(1 + 2 * a_H * \widehat{h}\right)} = Q,$$

which implies that $Q = a_H a_L$. This is a quadratic equation and we find that only one root is positive:

$$a_H = \frac{\widehat{h}Q + \sqrt{(\widehat{h}Q)^2 + (\widehat{h}+h)Q}}{(\widehat{h}+h)} \tag{21}$$

Using $Q = a_H a_L$ we determine a_L:

$$a_L = \frac{\sqrt{Q} * (\widehat{h}+h)}{\widehat{h} * \sqrt{Q} + \sqrt{\widehat{h}^2 Q + \widehat{h} + h}} \tag{22}$$

and,

$$\alpha_H = \frac{B - p_L \left(a_L^2 + Q\right)}{p_H Q + p_H (\widehat{h}+h) * a_H^2}. \tag{23}$$

8.2 Distortions in regime I_3

To determine the distortion of the output of low-cost contracts in regime I_3, we have to compare the output in the two cases a_L^{FB} and a_L^{SB} given by **??** from the main text and 6 from Appendix C respectively to see if the output is distorted:

$$
\begin{aligned}
a_L^{FB} &\lesseqgtr a_L^{SB} \\
\Leftrightarrow \sqrt{Qh} &\lesseqgtr \frac{\sqrt{Q} * (\widehat{h} + h)}{\widehat{h} * \sqrt{Q} + \sqrt{\widehat{h}^2 Q + \widehat{h} + h}} \\
\Leftrightarrow 0 &\lesseqgtr \widehat{h}\left(1 - 2\sqrt{Qh}\right)
\end{aligned}
$$

Since we know that $\widehat{h} > 0$, the question is reduced to:

$$
\begin{aligned}
0 &\lesseqgtr 1 - 2\sqrt{Qh} \\
\sqrt{Qh} &\lesseqgtr \frac{1}{2}
\end{aligned}
$$

This equals the first-best solution $a_L^{FB} = \sqrt{Qh}$. From the model of the unconstrained social planner we know that a_L^{FB} cannot be greater than $\frac{1}{2(1+\beta)}$ to ensure that the net social benefit of the contracts is positive. This means that

$$
\sqrt{Qh} < \frac{1}{2}
$$

By this we have shown that $a_L^{FB} < a_L^{SB}$, which means that the contract for the low-cost agent in regime I_3 is distorted upwards compared to the first-best for all feasible solutions.

8.3 The duration of regime I_3

In section X we argue that regime I_3 is optimal in a longer budget interval in the second-best case compared to the first-best case due to the information rent. This can be shown by calculating the difference between the two budget limits in each case:

$$
\begin{aligned}
B_3^{SB} - B_2^{SB} &> B_3^{FB} - B_2^{FB} \\
p_H\left(\left(\widehat{h} + h\right)\left(a_H^{SB}\right)^2 + Q\right) &> p_H\left(h\left(a_H^{FB}\right)^2 + Q\right) \\
\left(\widehat{h} + h\right)\left(\frac{\widehat{h}Q + \sqrt{(\widehat{h}Q)^2 + (\widehat{h} + h)Q}}{(\widehat{h} + h)}\right)^2 &> h\left(\sqrt{\frac{Q}{h}}\right)^2 \\
2\left(\widehat{h}Q\right)^2 + \left(\widehat{h}Q * \sqrt{(\widehat{h}Q)^2 + (\widehat{h} + h)Q}\right) &> 0
\end{aligned}
$$

The last equation is obviously true and therefore $B_3^{SB} - B_2^{SB} > B_3^{FB} - B_2^{FB}$ is also true. This means that the principal requires more money to engage all high-cost agents in the scheme in the second-best case.

9 Appendix C: The first-best case

In this appendix C, we present details of the solution to a first-best version of the social planner's problem in (1). As there exists symmetric information, the two participation constraints are binding:

$$\begin{aligned} s_L &\geq v_L(a_L) = a_L^2 + Q \\ s_H &\geq v_H(a_H) = ha_H^2 + Q, \end{aligned}$$

Thus, the social planner faces the following planning problem:

$$\begin{aligned} &\max\ W = \alpha_L p_L \left[a_L - (1+\beta)\left(a_L^2 + Q\right)\right] \\ &+\alpha_H p_H \left[a_H - (1+\beta)\left(ha_H^2 + Q\right)\right] \\ s.t.\ B \geq\ &\alpha_L p_L \left(a_L^2 + Q\right) + \alpha_H p_H \left(ha_H^2 + Q\right) \end{aligned} \tag{24}$$

9.1 Using only the low-cost agent ($\alpha_H = 0$)

The solution follows the same structure as the second-best solution. As long as $\alpha_H = 0$, there is no informational problem in either case and the solution is therefore identical to the one presented in the main model in equation 3, 4 and 5.

9.2 Using both agent types ($\alpha_H > 0$)

9.2.1 I_3: Rationing the high-cost agents ($\alpha_H < 1$)

In this regime, the high-cost agents are involved at their first-best contract type as far as the budget allows it. Using the FOC's the solution is:

$$a_L = \sqrt{Qh} \tag{25}$$

$$a_H = \sqrt{\frac{Q}{h}} \tag{26}$$

$$\alpha_H = \frac{B - p_H * (h+1)\,Q}{2 * p_H * Q} \tag{27}$$

9.3 I_4: Increasing procurement towards the unconstrained case ($\alpha_H = 1$)

When all agents are offered a contract, the social planner increases both contracts proportionally as:

$$a_L = h * a_H,$$

to the optimal level. Inserting into the budget constraint gives us:

$$a_L^2 = h^2 \frac{B-Q}{p_L h^2 + p_H h} \quad (28)$$

$$a_H^2 = \frac{B-Q}{p_L h^2 + p_H h} \quad (29)$$

However, this regime is only relevant until the point where the budget constraint is no longer binding, i.e. it no longer excludes the social planner from reaching the overall social optimum in the first-best situation.

9.4 The unconstrained solution

The social planner without a budget constraint can reach the socially optimal level of output. He maximize the objective function given in 24 and the solution is:

$$a_L = \frac{1}{2(1+\beta)} \quad (30)$$

$$a_H = \frac{1}{2h(1+\beta)} \quad (31)$$

The solution is similar to the second-best case but without the distortion of the information rent.

9.5 Budget limits

The limit between I_1 and I_2 is given by:

$$B_1 = 2p_L Q \quad (32)$$

The budget limit where the high-cost agents are involved in the procurement scheme is determined in the same way as in the main model:

$$B_2 = p_L (Qh + Q) = p_L (h+1) Q \quad (33)$$

The limit between I_3 and I_4 is the point in I_3 when $\alpha_H = 1$:

$$B_3 = p_L Q (h+1) + 2p_H Q = B_2 + 2p_H Q \quad (34)$$

The endpoint of the model is where the budget is no longer binding:

$$B_4 = \frac{1}{4(1+\beta)^2} \left(p_L + \frac{p_H}{h}\right) + Q \quad (35)$$