Spatial Proximity and Complementarities in the Trading of Tacit Knowledge^{*}

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Abstract

We model knowledge-trading coalitions in which the transfer of tacit knowledge is unverifiable and requires face-to-face contact, making spatial proximity important. When there are sufficient "complementarities" in knowledge exchange, successful exchange is facilitated if firms can meet in a central location, thereby economizing on travel costs. When complementarities are small, however, a central location may be undesirable because it is more vulnerable to cheating than a structure involving bilateral travel between firms. We believe that our framework may help explain the structure and stability of multimember technology trading coalitions such as Sematech and Silicon Valley.

Keywords: Tacit Knowledge, Clusters, Knowledge Trading, Complementarities, Spatial Proximity

JEL Codes: L1, O3, R1

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1 Introduction

It is widely recognized that the creation and dissemination of knowledge is central to modern economic growth, particularly in high-technology sectors such as computing, biotechnology, and telecommunications. How best to organize firms and industries to facilitate this process is a topic of ongoing research interest. In many industries, it is impractical for each firm to generate all relevant knowledge within its own vertically integrated structure, and the exchange of knowledge is central to industry success. This fact poses a serious organizational problem because knowledge is a difficult good on which to contract. It may be virtually impossible to specify in advance the nature of the knowledge to be exchanged, or to verify *ex post* whether the promised knowledge has in fact been delivered.¹ Contracting difficulties are especially severe for *tacit* knowledge, that is, know-how or skills that are embodied in human capital and difficult to codify.²

Tacit knowledge takes a variety of forms. In a manufacturing setting, learning-bydoing is critical in many industries. For example, the fabrication of silicon wafers is central to modern semiconductors, and is a delicate art that is only gradually learned on the job. Managerial processes more generally also involve tacit knowledge. While much has been written about total quality management, Womack (1991) points out that American automobile manufacturers took years to learn the process from the Japanese, and only began to develop mastery through joint ventures with Toyota and Honda.³ Critical to these and other examples is that sharing tacit knowledge requires face-to-face contact; reading about the skills involved is not sufficient.

When meetings are essential to knowledge exchange, spatial proximity plays a natural role in determining the cost of sharing knowledge. Marshall (1895) famously stressed that knowledge spillovers are a driving force for the agglomeration of industries. More recently, Saxenian (1994) and Porter (1998) have provided engaging

¹Aydogan (2002) studies empirically the governance structures used by Silicon Valley firms to support the transfer of knowledge.

²Polanyi (1958) provides the seminal account of tacit knowledge and its characteristics.

³An example of tacit knowledge within the economics profession is the art of model specification in econometrics. Welsch (1986, p. 405) puts matters bluntly: "Even with a vast arsenal of diagnostics, it is very hard to write down rules that can be used to guide a data analysis. So much is really subjective and subtle...A great deal of what we teach in applied statistics is *not* written down, let alone in a form suitable for formal encoding. It is just simply 'lore'."

accounts of the role of spatial clustering in creating regional economic advantages.⁴ There is also a growing empirical literature documenting the importance of spatial proximity for knowledge spillovers between firms. For example, Jaffe, Trajtenberg and Henderson (1993) find that patent citations are significantly more likely to come from within the same country, state, and even metropolitan area than would be predicted by the geographical dispersion of similar research. Audretsch and Feldman (1996) find that innovative activity, as measured by technological innovations actually introduced, tends to be more geographically clustered in industries that place greater reliance on research and development and on skilled labor.

There are several possible explanations for the observed importance of geographical clustering in knowledge-intensive industries. The most familiar of these is the notion of localized knowledge "spillovers," but this is more a description than an explanation.⁵ Underlying transmission mechanisms presumably rely upon interpersonal sharing of knowledge, which may occur through a variety of means, including casual conversation in bars after work, rapid employee turnover, and intentional meetings between employees of different firms. The survey work of Levin *et al.* (1987) documents the importance of interpersonal communication as a means for firms to acquire external knowledge. In particular, the use of publications and technical meetings, informal conversations, and hiring away employees from other firms are all important, and their use tends to be highly correlated. Leamer and Storper (2001) argue that proximity is an important source of competitiveness in large part because face-to-face meetings are necessary for the exchange of complex knowledge.

The foregoing work, while intriguing, has not presented formal models of the role of spatial proximity in knowledge-based industry clusters. Our goal is to develop a simple dynamic model in which competing firms trade tacit knowledge, the transfer of which is possible only when individuals meet face-to-face.⁶ As a result, geographical location becomes essential to the analysis. For simplicity, we do not model the research process, but rather assume that in each period each firm costlessly obtains knowledge that is valuable in achieving a cost reduction. This allows us to maintain

⁴According to Porter (1998, p. 77), "[c]lusters are geographical concentrations of interconnected companies and institutions in a particular field."

⁵De Bondt (1996) provides an interesting synthesis of the industrial organization literature on spillovers and their implications for innovative activity.

⁶Malmberg and Maskell (2001, p. 11) point out that "most well-known examples of industry agglomeration are obviously based on the horizontal dimension, since they are made up of several firms operating in the same industry."

a focus on spatial proximity and the process of knowledge exchange. We construct a Cournot quantity game in which each of N firms is represented by a single individual who must decide whether to travel to meetings with the other members of a coalition over time, with the distance between each pair of firms equal to d. In each period, firms decide whether to travel to meet with other member of the coalition, and if a meeting occurs, whether to truthfully reveal their own knowledge. We analyze the equilibrium strategies for a one-shot and a repeated Cournot output game. Following Eaton and Eswaran (1997), we consider expulsion from the coalition as the punishment mechanism in the repeated game.

We consider two possible organizational structures for exchange, a centralized meeting location and bilateral travel. The "central location" structure is possible only if a given firm's knowledge can be conveyed without being in the presence of the firm's actual facilities. This setting corresponds to the presence of a joint facility that can be shared by all firms, akin to the "foundry" model for semiconductor production.⁷ This setting also applies to an industry in which tacit knowledge is entirely independent of any physical facilities, and hence meetings can be held at a convenient centrally located hotel or conference center. The "bilateral travel" structure is relevant when a firm's tacit knowledge is intimately tied to its actual physical facilities. This is likely to be the case, for example, for benchmarking of complex manufacturing processes, e.g. total quality management (TQM), in which plant visits appeared to be necessary for Americans to truly comprehend the Japanese management approach.

The impact of knowledge exchange on costs depends on the extent to which the knowledge of the two parties is complementary. In strictly "independent" knowledge exchange, one firm can passively absorb the knowledge presented by another without revealing his own knowledge, and there are no joint gains from mutual exchange. In "complementary" knowledge exchange, there is an interaction that generates new insights and joint benefits not recognized by either party prior to the encounter, but only if both parties actively exchange their knowledge. In this type of exchange, both parties gain from a mutual revelation of information, and the whole is greater than the sum of the parts. We show that the balance between these two forms

⁷See National Research Council (2003) for further discussion of this model, especially the section on the Taiwanese industry.

of knowledge has important implications for the sustainability of knowledge trading. In particular, knowledge sharing may be an equilibrium—even in a one-shot game—if there is enough complementarity in the knowledge exchange process. In general, greater complementarities facilitate the exchange of knowledge across greater distances, thereby supporting the formation of successful clusters.

Interestingly, we find that the advantages of particular organizational structures may be related to the extent of complementarities in knowledge exchange. When complementarities are large, knowledge exchange is facilitated when firms have the ability to meet in a central location, thereby economizing on travel costs. When complementarities are small, however, a tradeoff emerges in the use of a central location. While the structure reduces travel costs, it is also more vulnerable to cheating than a structure involving bilateral travel between pairs of firms. With a central location, a firm can opportunistically cheat all other firms in the industry by traveling to the center, passively absorbing knowledge from all its rivals, and withholding its own knowledge. In the bilateral travel structure, however, a firm can only cheat a subset of the other firms in its industry before its cheating behavior is identified and punished. When rival firms visit the cheater before it visits them, rivals learn that the cheater is withholding its knowledge, and reciprocate by withholding their knowledge from it. This makes cheating less profitable than in the central location structure.

We are aware of only two other papers that attempt to formalize the role of spatial proximity in knowledge exchange. Cooper (2001) models information transmission via job mobility, and finds that contractual clauses restricting mobility are generally welfare-decreasing. Berliant et al. (2000) develop a model in which individuals search for others with complementary knowledge; they derive equilibria in which the extent of agglomeration is endogenously determined.⁸

In the following section, we present our basic model of knowledge exchange in an oligopoly, analyze the stage game involving travel to meetings, knowledge exchange, and output choice, and lay out the structure of the repeated game. In section 3, we study the knowledge-sharing equilibria that can be sustained when all firms travel

⁸Goyal and Moraga-Gonzalez (2001) adopt a social network approach, modeling in detail the formation of links between firms that allow for knowledge sharing, and assessing the equilibrium structure of such networks. Eaton and Eswaran (1997) study the performance of technology-trading coalitions that ostracize cheaters while the remaining members continue to cooperate. Neither of these papers considers the role of spatial proximity or knowledge complementarities, however.

to a central location. Section 4 studies the case where pairs of firms travel to one another's facilities to exchange knowledge; in both sections 3 and 4 we emphasize the relationship between knowledge complementarity and the sustainability of knowledge sharing. Section 5 concludes. All proofs are relegated to the Appendix.

2 Knowledge Exchange in an Oligopoly

In this section we lay out a simple model of knowledge exchange in an oligopoly. We focus on the case where all firms in the industry are involved in sharing information with one another.⁹ In each period, each firm generates some new tacit knowledge through a costless and unmodeled process. This knowledge can be transferred to others, but only through face-to-face contact. Furthermore, whether such knowledge was transferred cannot be verified by a court, and hence is non-contractible. Knowledge transmission requires that at least one party to the exchange travel to a meeting with the other party, incurring a travel cost dependent on the nature of the knowledge (as described in more detail below in the subsection on Travel) and the distance between them. At the meeting, each firm chooses whether to share its knowledge with the other. Let $x_{ij} \in \{0, 1\}$ be Firm *i*'s report to Firm *j*, where $x_{ij} = 1$ indicates sharing and $x_{ij} = 0$ indicates withholding. We will denote by **x** the array of the firms' choices.

As mentioned in the Introduction, we allow for two types of conversations during meetings. One type involves "independent" knowledge exchange: the listener simply absorbs what the speaker presents, and the content received is independent of any comments by the listener. The second type of exchange involves "complementarities," and is interactive and collaborative. In this type of interaction, there is a dynamic give and take in which new insights may be generated that were not

⁹Most papers on knowledge sharing assume that all firms within an industry are involved in knowledge sharing. For example, this is the assumption made by Kamien, Muller and Zang (1990) and Cooper (2001). Some papers, such as Morasch (1995), consider only two firms, so that if knowledge sharing occurs, it is necessarily done throughout the whole industry. Eaton and Eswaran (1997) point out via a numerical example that it is possible for multiple coalitions to emerge, though they do not present general conditions under which this is likely to occur. Their main thrust, however, is to show that cooperation is better supported by "stacked reversion" (in which a firm that "cheats" on its partners within a coalition is permanently ejected) than by "Nash reversion" (in which the coalition dissolves entirely if a firm cheats). We build on their insights and assume the punishment for cheating is ejection from, rather than dissolution of, the coalition.

recognized by either party prior to the meeting; co-authors on research papers are familiar with this phenomenon. We model these two types of exchange in different ways. Let each firm's unit cost function be $c_i(\mathbf{x}) = \alpha - \beta \sum_{j \neq i} x_{ji} - \gamma \sum_{j \neq i} x_{ij} x_{ji}$. Note that firm *i*'s own cost-reducing knowledge is already reflected in the parameter α . "Independent" exchange is captured through the second term, involving the coefficient β , while exchange involving complementaries is captured in the third term, involving coefficient γ . Exchange benefits from complementarities can only be realized if both firms involved share their knowledge. One way to think about the role of complementarities is to recognize them as an example of supermodularity.¹⁰ As we will see, the relative importance of these two types of exchange for cost reduction has interesting implications for the sustainability of knowledge trading.

In the remainder of this section we first present the basic structure of the stage game, and then discuss the repeated game.

2.1 The Structure of the Stage Game

We divide the stage game into three parts. First, firms decide whether to travel to meetings for the purpose of exchanging knowledge. Second, firms at a meeting decide whether to share their knowledge, which can generate cost reductions. Third, firms play a Cournot output game. As is standard in such games, we solve the three parts in reverse chronological order using backward induction.

2.1.1 The Output Game

There are N firms. They compete in a homogeneous-products Cournot oligopoly. Let q_i be the output chosen by firm i, and $Q = \sum_{i=1}^{N} q_i$ be total industry output. Demand is given by P(Q) = a - bQ. Define $Q_{-i} = Q - q_i$. Suppose $c_i(\mathbf{x}) = c$ for all $i \neq j$, while $c_j(\mathbf{x}) = c_j$ for firm j. Firm j's profit function is $\pi_j = (P(Q) - c_j)q_j$. It is straightforward to find reaction functions, and solve them for equilibrium output levels q_i^* and price P^* . If all firms but j are symmetric, then equilibrium profits are

$$\pi_i^* = b(q_i^*)^2 = \frac{(a - 2c + c_j)^2}{b(N+1)^2}$$

¹⁰The concept of supermodularity has been employed in a number of economic models. For example, Holmstrom and Milgrom (1994) exploit supermodularities to develop an economic theory of incentive structures within the firm.

$$\pi_j^* = b(q_j^*)^2 = \frac{[a - Nc_j + (N-1)c]^2}{b(N+1)^2}.$$

2.1.2 Knowledge Exchange

Given they can anticipate the results of Cournot competition, firms must decide whether to participate in knowledge exchange in each of their pairwise encounters with other firms. We analyze a representative meeting, assuming that one firm or the other has already traveled to the meeting place. In the following subsection, we study the firms' decisions whether or not to travel.

Let us suppose that in each period, all firms meet with all other members of the industry. Since the firms are symmetric, a given firm will treat all other firms the same with regard to its decision whether to share its knowledge. In other words, $x_{ij} = x_{ik}$, for all $j \neq k$. From our preceding work, we know that if all firms participate in knowledge exchange, then each has unit cost $c^{coop} \equiv \alpha - (\beta + \gamma)(N - 1)$. If all firms but j exchange knowledge, and j withholds its knowledge from all others in the industry, then $c_i = \alpha - (\beta + \gamma)(N - 2)$ for all $i \neq j$ and $c_j = \alpha - \beta(N - 1)$. We will say that complementarities are "large" if $c^{coop} < c_j$.

Because the exchange of tacit knowledge is non-contractible, conventional wisdom suggests that exchange cannot succeed in a one-shot setting, although it may be possible in a repeated context. Indeed, this is the case in the model of Eaton and Eswaran (1997). The following lemma identifies the Nash equilibria in the subgame involving knowledge sharing and Cournot competition.

Lemma 1: Suppose all firms engage in meetings with one another. If knowledge complementarities are small, i.e. $\gamma < \beta/(N-1)$, then the only Nash equilibrium for the remainder of the stage game is for all firms to withhold their knowledge. If knowledge complementarities are large, i.e. if $\gamma > \beta/(N-1)$, then there are two Nash equilibria, one with no knowledge sharing and one with knowledge sharing by all firms.

Proof: See the Appendix.

It is instructive to compare this result to the argument of Arora (1996), who

and

also shows that the exchange of tacit knowledge can be supported in a one-shot game, but only if it can be tied contractually to the exchange of another input, such as hardware, whose transfer is verifiable. Our analysis differs from his in that we make no use of verifiable inputs nor of formal contracts. We are concerned with a setting where both parties possess tacit knowledge that is of value to the other party, and where the complementarity is an innate part of the knowledge the parties possess. Indeed, we find that when such complementarities are present, a formal contract for knowledge exchange is unnecessary if complementarities are strong enough. Nevertheless, there still exists a Pareto-dominated equilibrium in which no sharing takes place, since research complementarities confer no benefits on the sharing firm if rivals are withholding knowledge.¹¹

2.1.3 Travel

We turn next to the firms' decisions whether to travel to meetings for the purpose of knowledge exchange. Our specification of travel costs makes use of the "spokes" model of location proposed by Chen and Riordan (2003), which generalizes the standard Hotelling model of two firms at opposite ends of a line segment. Imagine N firms spread evenly around a circle of diameter d, each of which is connected to the "hub" of the wheel by a "spoke" of length d/2. Two firms can agree to meet at a "central location," namely the hub of the wheel, in which case each firm incurs travel cost d/2, normalizing the unit cost of travel to one. Travel by firm *i* to the facilities of firm *j* requires traveling along the spoke connecting firm *i* to the hub, and then along the spoke that connects the hub to firm *j*, for a total travel distance of *d*. Thus, in this model, all firms are equidistant from one another.¹²

The amount of travel required for knowledge exchange depends upon the nature of the knowledge involved. We consider two organizational structures which we denote

¹¹The sharing equilibrium may be more "reasonable" than not sharing when the complementarity parameter, γ , is large relative to β . For a two-person game, one can show that the sharing equilibrium is risk-dominant when γ is large relative to β . (See Harsanyi and Selten (1988) for development of the concept of risk dominance.) Intuitively, this means that sharing is preferred even when one is uncertain what one's rival will play. Generalizing the notion of risk dominance to our *N*-person game would require the use of combinatorial techniques, however, and we leave this for future research. The point we make here is simply that the sharing equilibrium can be justified as reasonable, even in a one-shot game, under certain conditions.

¹²For simplicity, we normalize d to be the round-trip cost of travel.

using the variable θ : 1) Exchange at a central location ($\theta = C$), and 2) Exchange that must be conducted through bilateral travel because tacit knowledge is embodied in location-specific processes and structures ($\theta = B$). The first of these structures corresponds roughly to Markusen's (1996) notion of a "hub-and-spoke" structure, which is associated with the organization of the automobile and bio-pharmaceutical industries, or a "state-anchored district," in which a key research facility, government laboratory, military base or state-supported university serves as the central hub. The second structure is closer to the traditional Marshallian industrial district often associated with Silicon Valley and the "Third Italy."¹³ The notion that tacit knowledge is embodied in products, processes and practices is commonly acknowledged by scholars of innovation and industrial clusters.¹⁴ However, to the best of our knowledge, our paper is first to formally model the implications of this distinction for the spatial organization of industry.

We will represent the per-firm travel costs in each regime by $T(d, \theta)$. The relationship between knowledge regime and travel cost is as follows. In the case of travel to a central location, all firms can meet at the hub, each incurring travel costs T(d, C) = d/2. Once they are at the hub, the firms' representatives engage in a series of pairwise meetings that have no additional marginal cost. In the case of bilateral travel, full cooperation requires that each firm must travel to all other firms, for total travel cost of T(d, B) = d(N - 1).¹⁵

Note that the results of Lemma 1 concern the behavior of firms once they are in a meeting with another firm. Further analysis is needed to determine whether firms have incentives to incur the costs of travelling to a meeting for purposes of knowledge exchange. In a one-shot game, firms will never travel if $\gamma < \beta/(N-1)$, since the unique equilibrium outcome for meetings is for both firms to withhold knowledge. When $\gamma > \beta/(N-1)$, however, travel may be profitable.

Throughout the paper we are interested in when knowledge-sharing outcomes can be supported as non-cooperative equilibria. One of the complexities that arise when

¹³We thank an anonymous referee for suggesting that we consider the central location structure.

¹⁴See Lipsey (2002) for a detailed discussion of alternative forms of embodied technological knowledge.

¹⁵We have also analyzed the case where it is sufficient for one firm in a pair to travel to the other firm's facilities, and both firms can exchange knowledge at this one meeting. The results are similar to the case of bilateral travel, however, and since we felt bilateral travel is more common in practice, we have streamlined the paper by eliminating the discussion of one-way travel.

spatial proximity is included in a model of knowledge exchange is that one must address explicitly the multiple ways in which firms can deviate from a sharing agreement. In particular, there are two dimensions on which deviation can occur: whether a firm travels to other firms when it is expected to, and whether a firm shares knowledge once it is in a meeting with another firm. We will use the notation "S" to indicate that a firm shares knowledge, and "W" to indicate that it withholds knowledge; we will let "T" indicate that a firm travels, and "NT" indicate that it does not. With a central location, there are four possible strategies: "S/T", "S/NT", "W/T", and "W/NT." Clearly, "S/T" is the fully cooperative outcome. We will denote a given exchange strategy in the case of a central location by $\sigma \in \{S/T, S/NT, W/T, W/NT\}$. (The strategies for bilateral travel are more complex, and are described in section 4.) Which of the possible deviations is most profitable will depend on various parameters of the model, as we will see below.

In the remainder of the paper we will be concerned with identifying the maximum distance across which firms can be located and still support knowledge exchange. We will use the notation d^{σ}_{θ} to be the "one-shot distance threshold" for knowledge exchange, that is, the maximum distance that will support knowledge exchange in a one-shot game in organizational structure θ with exchange strategy σ , where $\theta = C$ indicates the use of a central location, and $\theta = B$ indicates bilateral travel. Similarly, we will let D^{σ}_{θ} be the "repeated game distance threshold," that is, the maximum distance that will support knowledge exchange in a repeated game in structure $\theta \in$ $\{C, B\}$ and exchange strategy σ . For both the one-shot and repeated games, we will use the notations d_{θ} and D_{θ} without a superscript to indicate the minimum distance threshold when firms play their optimal strategies.

2.2 Knowledge Exchange in a Repeated Game

In studying the repeated game, we will emphasize the case where $\gamma < \beta/(N-1)$, since cooperation is sustainable even in the one-shot game otherwise. When complementarities are small, that is if $\gamma < \beta/(N-1)$, knowledge exchange will not occur in a one-shot game but exchange may nevertheless be possible in an ongoing trading relationship. In a repeated setting, with discount factor δ , the net present value of cooperating forever is simply

$$V^{coop}(d,\theta) = [\pi^{coop} - T(d,\theta)]/(1-\delta).$$

If all firms fully cooperate in knowledge exchange, that is, they travel to meetings and share knowledge at meetings, then each firm has unit cost $c^{coop} = \alpha - (\beta + \gamma)(N-1)$ and the cooperative payoff in the stage game (gross of travel costs) is

$$\pi^{coop} = \frac{[a - \alpha + (\beta + \gamma)(N - 1)]^2}{b(N + 1)^2}$$

We will assume that if firm j cheats on a knowledge sharing agreement today, he will be detected within one period, and will be ostracized by the rest of the industry forever after. Eaton and Eswaran (1997) refer to this form of punishment as "stacked reversion," and explore in detail the technical aspects of its use as a means of supporting cooperation in a supergame. The potential concern is that once a single defector is ostracized, the remaining coalition members might have incentives to defect sequentially, thereby causing the coalition to break down. If this is the case, then the only sustainable punishment mechanism is the threat that the entire coalition will be dissolved ("Nash reversion") if one member ever cheats.¹⁶ Eaton and Eswaran conclude that when costs fall linearly with the number of members in a coalition, as in our case, there exists a critical discount factor such that stacked reversion is viable for discount factors above this critical level. They are unable to provide an analytical result, but reach the conclusion by using Mathematica to derive a power series approximation for the critical discount factor, and then finding that the approximation is increasing in the size of the coalition. For our case, which includes knowledge complementarities and spatial proximity, it is difficult to determine how the power series approximation changes with N. Thus, we focus on the case where δ is large.

When ostracism is the punishment imposed by the cooperating firms on a "cheater," we can specify the punishment payoff as follows. If firm j is caught cheating and ostracized, then firm j's cost in the future will be $c_j = \alpha$, while the other firms in the

¹⁶We thank an anonymous referee for pointing out that an interesting extension of the analysis would allow for sequential knowledge trades each period. In this case, as long as knowledge were not embodied in a particular firm's facilities, then each successive firm would be able to pass along its knowledge, along with that of all the other previous firms. The firm that is the last to trade could extract all the information from all other industry members through a single act of cheating. This would reduce the cost of cheating, and make knowledge sharing more difficult to sustain. While this is an intriguing notion, it raises questions that go beyond the scope of the present paper.

industry have costs $c_i = \alpha - (\beta + \gamma)(N - 2)$. Define

$$\widetilde{c} = \alpha + (\beta + \gamma)(N - 1)(N - 2).$$

Then profits for a firm that is ostracized, while the rest of the industry cooperates, are

$$\pi^{ostracize} = \frac{[a-\widetilde{c}]^2}{b(N+1)^2}.$$
(1)

Throughout the remainder of the paper we will assume that an ostracized firm continues to have strictly positive output and profits, and is not forced to exit the industry. For future reference, it is worth noting that

$$\pi^{coop} = \frac{[a - \tilde{c} + (\beta + \gamma)(N - 1)^2]^2}{b(N + 1)^2}.$$
(2)

3 Central Location

In this section we study the case where tacit knowledge is exchanged at a single central location. This structure can encompass a variety of different situations. First, it can represent a situation where tacit knowledge takes the form of pure human capital, and hence is independent of the facilities of the organization within which it That is, its transmission is not tied to the use or availability of particular exists. pieces of equipment. Thus, while industry members need to engage in face-to-face meetings, the location for such meetings need not be the facilities of an industry member. A hotel or convention center is adequate for meetings involving this form of knowledge. As a result, it is feasible for all industry members to travel to a central location to exchange knowledge. Second, one can think of the analysis in this section as applying to an industry with Markusen's (1996) "hub-and-spoke" structure or her "state-anchored district" structure. In either of these structures, there is a central facility that serves as a common hub to which all firms travel in order to exchange knowledge. A university can serve as such a central location. Alternatively, as Murphy (1991) explains, the central location could be a non-university facility, as when the Microelectronics and Computer Cooperative (MCC) created a joint research lab, and built it in a location where it would not advantage any member over others for accessing knowledge from the lab. A somewhat similar situation emerged in Taiwan, through the notion of a "foundry" for the production of semiconductors. In 1987, the Taiwanese government provided substantial funding to create the Taiwan Semiconductor Manufacturing Company (TSMC), which served as a central production facility that could be utilized by a large number of smaller "fab-less" design companies, who were thereby freed from the need to invest in their own manufacturing facilities.¹⁷

3.1 Equilibria in the Stage Game

If $\gamma < \beta/(N-1)$, then from Lemma 1 it cannot possibly be profitable to travel in a one-shot game, since firms will withhold knowledge at meetings. Assuming $\gamma > \beta/(N-1)$, however, then if a firm travels to the central location, it will share knowledge with all other firms present. Its travel costs are T(d,C) = d/2. Then the cooperative payoff is $\pi^{coop} - d/2$. Alternatively, if the firm chooses not to travel, it obtains no knowledge from any other firm. We will suppose that all other firms continue to cooperate. Then if firm j chooses not to travel, its cost will be $c_j = \alpha$, while the other firms in the industry have costs $c_i = \alpha - (\beta + \gamma)(N-2)$. Note that this is exactly the same as the cost configuration obtained when firm j is ostracized, so profits for a firm that chooses not to travel, while the rest of the industry cooperates, are simply $\pi^{ostracize}$.

It is straightforward to define the threshold distance $d_C(a, b, N, \alpha, \beta, \gamma)$ such that in the one-shot game travel is profitable for distances below the threshold. Suppressing the dependence of d_C on various parameters we can write:

$$d_C = 2 \left[\pi^{coop} - \pi^{ostracize} \right]. \tag{3}$$

Thus, the exchange of tacit knowledge is an equilibrium even in a one-shot game with strictly positive travel costs, as long as there is sufficient complementarity to the knowledge being shared and travel costs are not too great. We summarize our analysis of the stage game in the following Lemma.

Lemma 2: When knowledge exchange is conducted at a central location, if complementarities are small, i.e. $\gamma < \beta/(N-1)$, then the unique equilibrium of the stage game is that no travel and no knowledge sharing occurs. If complementarities are

¹⁷National Research Council (2003), pp. 149-160.

large, however, i.e. $\gamma > \beta/(N-1)$, then knowledge exchange with travel to a central location is an equilibrium in the stage game if $d \leq d_C$.

We can gain further insight into the determinants of the travel threshold by expanding the expression for d_C , which yields

$$d_C = \frac{2(\beta + \gamma)(N-1)^2}{b(N+1)^2} [2(a-\alpha) - (\beta + \gamma)(N-1)(N-3)].$$

Differentiation allows us to identify how changes in the underlying parameters of the model affect the distance across which knowledge exchange can be supported. Outward shifts of the the demand curve, i.e. increases in parameter a, increase the threshold distance for knowledge exchange. Similarly, reductions in production $\cos \alpha$ expand the distance across which knowledge can be exchanged. In addition, when the demand curve becomes steeper, i.e. b decreases, the threshold distance Furthermore, when the cost-reduction parameters β or γ increase the increases. threshold distance also increases, as long as $N \leq 2 + \sqrt{(a-\alpha)/(\beta+\gamma)}$. Speaking broadly, these comparative statics indicate that firms can cooperate over greater distances when the available market surplus—either through expanded demand or reduced cost—is greater. Note that in this case the parameters β and γ have identical effects on the distance threshold, but this will not be true for the repeated game. The effect of N on the distance threshold is more complex; it increases with N up to a point, and then decreases.

3.2 Equilibria in the Repeated Game

In the supergame, a firm has the choice to cooperate forever, thereby earning payoff

$$V^{coop}(d, C) = [\pi^{coop} - d/2]/(1 - \delta).$$

Alternatively, a firm can decide to cheat on the sharing agreement. If $\gamma > \beta/(N-1)$, then Lemma 1 shows that a firm will share knowledge if it travels to the central location, so cheating can only be accomplished by not traveling in period t, which ensures that the firm will be ostracized from period t+1 onwards. This yields the payoff

$$V^{cheat} = \pi^{ostracize} + \delta \frac{\pi^{ostracize}}{1-\delta} = \frac{\pi^{ostracize}}{1-\delta}.$$

If $\gamma < \beta/(N-1)$, there is an alternative cheating strategy available in which a firm travels to the central location but does not share knowledge. In this case, a firm that decides to deviate from full cooperation will withhold knowledge at the meeting, since it will be ostracized by other firms in the future and it is not worthwhile to share information in the present period. If all firms but j fully cooperate, then $c_i = \alpha - (\beta + \gamma)(N-2)$ for all $i \neq j$. Firm j, however, takes advantage of the knowledge shared by all N-1 other firms, leaving it with cost $c_j = \alpha - \beta(N-1)$. Gross of travel cost, the one-period profit for firm j from a strategy of "withhold but travel" is

$$\pi^{W/T} = \frac{[a - \tilde{c} + \beta N(N-1)]^2}{b(N+1)^2}.$$
(4)

It is easy to see that $\pi^{W/T} > \pi^{ostracize}$; in addition, when $\gamma < \beta/(N-1)$ it is also the case that $\pi^{W/T} > \pi^{coop}$. With this cheating strategy, the firm's discounted present payoff is

$$V_C^{W/T} = \pi^{W/T} - \frac{d}{2} + \delta \frac{\pi^{ostracize}}{1-\delta}.$$

We characterize the threshold distance below which knowledge exchange can be sustained in the following proposition.

Proposition 1: For travel to a central location, knowledge exchange is an equilibrium in the repeated game if

$$d < D_C = \begin{cases} d_C \equiv 2 \left[\pi^{coop} - \pi^{ostracize} \right] & \text{if } \gamma \ge \beta/(N-1) \\ D_C^{W/T} \equiv \frac{2}{\delta} \left[\pi^{coop} - \delta \pi^{ostracize} - (1-\delta)\pi^{W/T} \right] & \text{if } \gamma < \beta/(N-1) \end{cases}$$

Proof: See the Appendix.

Proposition 1 shows that the magnitude of complementarities is critical to determining the spatial distance across which knowledge exchange can be supported, since there is a kink in the expression for the distance threshold at the point where $\gamma = \beta/(N-1)$. Note that at $\gamma = \beta/(N-1)$ we have $\pi^{coop} = \pi^{W/T}$, so $D_C^{W/T} = d_C$. For $\gamma < \beta/(N-1)$, however, $\pi^{coop} < \pi^{W/T}$, which implies that $D_C^{W/T} < d_C$. Thus, the distance threshold increases for $\gamma > \beta/(N-1)$. The following Proposition considers in more detail how D_C is affected by changes in β and γ . **Proposition 2:** For travel to a central location, $\partial D_C / \partial \gamma \geq \partial D_C / \partial \beta > 0$, with $\partial D_C / \partial \gamma > \partial D_C / \partial \beta$ for $\gamma < \beta / (N-1)$. Furthermore, both $\partial D_C / \partial \gamma$ and $\partial D_C / \partial \beta$ are greater for $\gamma < \beta / (N-1)$ than for $\gamma > \beta / (N-1)$.

Proof: See the Appendix.

Proposition 2 establishes several interesting facts about the case of travel to a central location. Most importantly, knowledge exchange is sustainable across greater distances when it is more effective at reducing costs, whether knowledge exchange is of the independent type or the complementary type. However, the distance threshold is more responsive to increases in complementarities than to increases in the value of independent knowledge. Finally, the distance threshold is concave in both β and γ . The relation between γ and D_C is illustrated in Figure 1.

[Insert Figure 1 here]

4 Bilateral Travel

We turn now to the case of tacit knowledge that is intimately tied to the processes and equipment used at particular firms' facilities. This is the case for the types of process improvements for which firms like to benchmark their own progress.¹⁸ It would be the case, for example, for total quality management, which took American auto makers a decade to learn from the Japanese. In such settings, collective routines, conventions, procedures or key leaders within firms may become essential repositories of knowledge. Visits to the facilities of other firms become important when knowledge is tacit, since a firm's representative can only communicate his tacit knowledge effectively on the premises of his own firm. Thus, successful knowledge exchange requires that both firms travel to one another. As a result, full cooperation requires that firms incur per-period travel costs T(d, B) = d(N-1).

In this setting, knowledge complementarities cannot emerge until both firms in a pair have visited each others' facilities. We will assume that within each stage, plant visits occur sequentially, and that if firm i first makes a visit to firm j, it is able to tell immediately upon leaving whether or not firm j has shared its knowledge. Firm

¹⁸For an introduction to the practice of benchmarking, see Jacobson and Hillkirk (1986) or Economist Intelligence Unit (1993).

i can thus condition its sharing strategy with firm j on firm j's sharing behavior. Complementarities only develop if both firms in sequence share their knowledge with one another. We will assume that firms in each pairwise combination are randomly selected to travel first.

4.1 Equilibria in the Stage Game

If a firm decides to cheat on the sharing agreement, it can do so by either not traveling, not sharing, or both. Furthermore, it may adopt a different strategy when it is the first firm in a pair to travel as opposed to the second. Thus, we can characterize a strategy in the stage game by a four-tuple expressing whether or not the firm will travel and/or subsequently share knowledge when it is the first traveler, and whether it will share knowledge and/or travel when it is the second traveler. Let $t_{ij}^1 \in \{T/NT\}$ be firm i's decision whether to travel to firm j if firm i is first to travel, while $t_{ij}^2 \in \{T/NT\}$ is firm i's decision whether to travel to firm j if firm i is second to travel. Similarly, $x_{ij}^1 \in \{S/W\}$ is firm i's decision whether to share or withhold knowledge with firm j if firm i travels first, while $x_{ij}^2 \in \{S/W\}$ is firm i's decision whether to share or withhold knowledge with firm j if firm i is second to travel. Note that x_{ij}^1 can be conditioned upon firm j's previous sharing behavior in the stage game, x_{ji}^2 ; of course, x_{ij}^1 is trivially conditional upon firm j's decision to travel, t_{ji}^2 , since *i*'s willingness to share is irrelevant if *j* does not travel. Similarly, x_{ij}^2 is trivially conditional upon firm j's previous travel behavior in the stage game, t_{ji}^1 . Finally, t_{ij}^2 can be meaningfully conditioned upon firm j's previous travel behavior in the stage game, t_{ji}^1 . In general, then, a strategy for firm *i* with regard to firm *j* can be expressed as $(t_{ij}^1, x_{ij}^1(x_{ji}^2); x_{ij}^2, t_{ij}^2(t_{ji}^1))$. The strategy (T, S(S); S, T(T)) is thus the fully cooperative strategy in which the firm shares knowledge and travels regardless of whether it moves first or second, as long as the other firm is also playing the cooperative strategy. In what follows, we suppress the dependence of x_{ij}^1 on x_{ji}^2 and t_{ij}^2 on t_{ji}^1 to keep our notation as simple as possible.

As should be apparent, a general analysis of strategic interaction in the bilateral travel game is potentially extremely complex, since there are sixty-four possible strategies for each firm in the stage game. Fortunately, most of them can be eliminated from consideration on the grounds that they are either dominated or cannot be part of any subgame perfect equilibrium. For example, if $t_{ij}^1 = NT$, then $x_{ij}^1 = S$ is dominated because firm *i* has no incentive to share knowledge once it has already destroyed any chance for achieving complementarities by refusing to travel. Similarly, if $x_{ij}^2 = W$, then $t_{ij}^2 = T$ cannot be part of a subgame-perfect equilibrium because firm j has incentives to withhold knowledge once firm i has destroyed any chance for achieving complementarities by refusing to share. Finally, if $x_{ij}^2 = S$, then $t_{ij}^2 = NT$ is sub-optimal because the only reason for firm i to share at first is that it intends to travel in the second half of the period, in order to obtain complementarity benefits. In addition, it is possible to eliminate some asymmetric strategies. For example, the strategies (T, W; S, T) and (T, S; W, T) cannot possibly be optimal; if complementarities are valuable enough to support sharing when i travels first, they should also support sharing when i travels second, and conversely.

Proceeding with the foregoing logic, we can winnow down the set of internally consistent potential equilibrium strategies to five: (T, S; S, T), (NT, W; W, NT), (T, W; W, NT), (T, S; W, NT), and (NT, W; S, T). The first represents full cooperation, and the second represents full non-cooperation. The third strategy takes advantage of the sequential nature of travel in this game, and involves "opportunistic plant visits" by firm *i*, in which firm *i* travels to firm *j* and absorbs knowledge, but withholds knowledge when *j* visits. The fourth and fifth strategies generate equal payoffs, and involve "selective sharing" in which firm *i* shares with half of its rivals and withholds from the other half; such strategies could potentially be optimal if the benefits of sharing with more firms increased more slowly than the (linear) costs of traveling.

If $\gamma < \beta/(N-1)$, then even if firm *i* shares its knowledge, firm *j* will not share if it is visited by firm *i*. Knowing this, neither firm will share; nor will either firm travel. The unique equilibrium strategy in the stage game is thus (NT, W; W, NT)for all firms.

If $\gamma > \beta/(N-1)$, then (NT, W; W, NT) remains an equilibrium, with payoff

$$\pi^{NT,W;W,NT} = \pi^{ostracize}.$$

Other strategies must be considered as possible equilibria, also. If all firms fully cooperate in knowledge exchange, that is, they travel to meetings and share knowledge at meetings, then each firm has unit cost $c^{coop} = \alpha - (\beta + \gamma)(N-1)$ and the cooperative payoff in the stage game is $\pi^{T,S;S,T} = \pi^{coop}$. Strategy (T,W;W,NT) implies that the firm obtains no complementarities, but engages in opportunistic plant vists that take knowledge from half of its partners, thereby achieving cost $c = \alpha - \beta(N-1)/2$ while its cooperative rivals have cost $c = \alpha - (\beta + \gamma)(N-2)$. It must travel to half

its partners, thereby incurring travel costs d(N-1)/2 and earning a payoff (gross of travel costs) of

$$\pi^{T,W;W,NT} = \frac{[a - \tilde{c} + \beta N(N-1)/2]^2}{b(N+1)^2}.$$

Finally, the "selective sharing" strategy, which can be implemented as (T, S; W, NT)or (NT, W; S, T) implies that the firm obtains complementarities with half of its rivals, but exchanges no knowledge with the other half. As in the previous strategy, the firm travels to half of its partners. If all firms but j fully cooperate, then firm j's cost is $c_j = \alpha - (\beta + \gamma)(N-1)/2$. Half of the other firms have $\cot \alpha - (\beta + \gamma)(N-1)$ and half have $\cot \alpha - (\beta + \gamma)(N-2)$. Computing the equilibrium for this setting is tedious due to the existence of three different groups of firms, each with different costs. Nevertheless, it can be shown that equilibrium profits for firm j in this case (gross of travel costs) are

$$\pi^{T,S;W,NT} = \frac{[a - \tilde{c} + (\beta + \gamma)(N - 1)^2/2]^2}{b(N + 1)^2}$$

Note that if $\gamma > \beta/(N-1)$ then $\pi^{T,S;W,NT} > \pi^{T,W;W,NT}$, so if a "cheater" engages in any travel, he also finds it optimal to share information with the firm to whom it travels. Still, there remains the question of whether to cheat using strategy (T, S; W, NT) or the strategy of total non-cooperation, (NT, W; W, NT). The following Lemma addresses this question.

Lemma 3: In the stage game with bilateral travel, the strategy (T, S; W, NT) can never be optimal.

Proof: See the Appendix.

The intuition behind Lemma 3 is roughly that the benefits of sharing increase as the square of the number of firms with which one shares, while the travel cost of sharing increases only linearly with the number of firms. Hence, if it is worthwhile to share selectively with half of one's rivals, it must also be worthwhile to share with all other firms. Although the strategy of selective sharing is internally consistent, the payoff structure of the game makes selective sharing a dominated strategy in the one-shot game. We can now characterize the amount of travel that can be supported in the stage game in the case of bilateral travel as contrasted with the case of travel to a central location.

Proposition 3 $d_B \leq d_C$. **Proof:** See the Appendix.

In the stage game, knowledge exchange can be sustained over greater distance for the case of travel to a central location. In both structures, the only feasible cheating strategy in the stage game is total non-cooperation, e.g., withholding all knowledge and not traveling. Although bilateral travel creates the possibility of a wide variety of "cheating" strategies, it turns out that in the stage game all but total non-cooperation are sub-optimal. Opportunistic plant visits cannot be effective in a one-shot game with symmetric firms, since withholding knowledge is only optimal if $\gamma < \beta/(N-1)$, in which case all firms refuse to share knowledge with others. Selective sharing cannot be profitable either; if $\gamma > \beta/(N-1)$, a firm is better off to either not travel at all or to fully cooperate, rather than to share with only half of the other firms. Thus, the payoffs from cooperation and from cheating are the same across the central location structure and the bilateral travel structure, and the only relevant question is the travel cost associated with cooperation in each case. It is easy to see that traveling to a central location requires less total travel than does bilateral travel between each pair of partners, so the use of a central location supports knowledge exchange over a greater distance in the stage game.

4.2 Knowledge Exchange in the Repeated Game

When complementarities are large, that is, when $\gamma > \beta/(N-1)$, knowledge exchange in a repeated game occurs under the same circumstances as in the stage game. Since selective sharing is not worthwhile, firms either cooperate fully or not at all. Furthermore, since total non-cooperation yields the payoff $\pi^{ostracize}$, there is no short-term gain to "cheating" on the agreement.

When complementarities are small, that is, when $\gamma < \beta/(N-1)$, knowledge exchange never occurs in a one-shot game but exchange may nevertheless be possible in an ongoing trading relationship. Now, however, we must consider the possibility of a short-term gain to cheating on the agreement, which may undermine cooperation. (Recall that in the central location structure, when complementarities are small the cheating strategy of traveling but withholding is more profitable than full cooperation.) Full cooperation requires both traveling to meetings and sharing knowledge at meetings, thereby earning payoff

$$V^{coop}(d,B) = \frac{\pi^{coop} - d(N-1)}{1-\delta}.$$

Alternatively, a firm can decide to cheat on the sharing agreement. The net present value of cheating is then given by

$$V_B^{\sigma} = (\pi^{\sigma} - T^{\sigma}) + \frac{\delta}{1 - \delta} \pi^{ostracize}, \qquad (5)$$

where $\sigma \in \{(T, W; W, NT), (NT, W; W, NT), (T, S; W, NT), (NT, W; S, T)\}$ indicates the firm's cheating strategy, π^{σ} is the firm's first-period payoff from strategy σ , gross of travel cost, and T^{σ} is the first-period travel cost associated with that strategy. Note that $T^{T,W;W,NT} = T^{T,S;W,NT} = T^{NT,W;S,T} = d(N-1)/2$, and $T^{NT,W;W,NT} = 0$. The firm finds it profitable to cheat if

$$(\pi^{\sigma} - T^{\sigma}) - [\pi^{coop} - d(N-1)] > \frac{\delta}{1-\delta} \left[\pi^{coop} - d(N-1) - \pi^{ostracize}\right]$$

By rewriting this expression, we can identify the maximum distance a firm is willing to travel per period to sustain cooperation in this model, relative to cheating strategy σ . Denote this distance by D_B^{σ} . Note that the most profitable cheating strategy depends upon the magnitude of complementarities. In particular, since we are concerned with the case of $\gamma < \beta/(N-1)$, we know that a cheater prefers strategy (T,W;W,NT) to strategy (T,S;W,NT) and to the equivalent strategy (NT,W;S,T). Thus we can limit our consideration of short-term cheating strategies in this case to (T,W;W,NT). The distance thresholds associated with this strategy and with total non-cooperation are

$$D_B^{T,W;W,NT} = \frac{2}{(N-1)(1+\delta)} \left[\pi^{coop} - \delta \pi^{ostracize} - (1-\delta) \pi^{T,W;W,NT} \right],$$

and

$$D_B^{NT,W;W,NT} = \frac{1}{N-1} \left[\pi^{coop} - \pi^{ostracize} \right] = \frac{d_C}{2(N-1)}.$$

As we have noted, when complementarities are small, firms may have incentives to engage in opportunistic plant visits and then refuse to reciprocate with knowledge disclosure when other firms visit. This raises the question whether in some circumstances knowledge exchange can be better supported when firms are located far enough apart to discourage opportunistic plant visits. Define $d^{T/W}$ as the distance beyond which a firm prefers total non-cooperation to a strategy of opportunistic plant visits, (T, W; W, NT). Then

$$d^{T/W} = \frac{2}{N-1} \left[\pi^{T,W;W,NT} - \pi^{ostracize} \right] = \frac{(N-1)(\beta+\gamma)}{b(N+1)^2} \left[2(a-\tilde{c}) + \beta N(N-1)/2 \right].$$

Is it possible that knowledge exchange can be supported for some $d > d^{T/W}$ but not for $d < d^{T/W}$? This question can most easily be addressed graphically. Figure 2 shows two possible configurations of the relevant payoffs and their relationship to d when $\gamma < \beta/(N-1)$. In both configurations, $V^{ostracize}$ is flat since it represents the payoff from total non-cooperation, which involves no travel at all. The full cooperation payoff is V^{coop} , which has slope $-(N-1)/(1-\delta)$. As discussed above, these two curves intersect at distance $D_B^{NT,W;W,NT} = d_C/(N-1)$. Because $\gamma < \gamma$ $\beta/(N-1)$, the relevant cheating payoff is $V^{T,W;W,NT}$. Since it involves one period in which the cheater travels to half of the other firms, followed by subsequent periods with no travel, it has slope -(N-1)/2, and is hence much flatter than V^{coop} . In Figure 2A, parameter values are such that $d^{T/W} < d_C/(N-1)$, so cooperation can indeed be supported for values of $d > d^{T/W}$. However, cooperation can also be supported for $d < d^{T/W}$, since $V^{coop} > V^{T,W;W,NT}$ for all $d < d_C/(N-1)$. Figure 2B represents a different set of parameters, for which $V^{T,W;W,NT} > V^{coop}$ for some $d < d_C/(N-1)$. Now, however, $d^{T/W} > d_C/(N-1)$ and cooperation can only be supported for $d \leq D_B^{T,W;W,NT} < d_C/(N-1) < d^{T/W}$. Thus, in either case, it is impossible for cooperation to be supported at some distance $d > d^{T/W}$ but not also for all $d < d^{T/W}$.

[Insert Figures 2A and 2B Here]

What we have shown is that it is never the case that cooperation fails at shorter

distances but succeeds across greater distances.¹⁹ We state this result in the following Lemma, a formal proof of which is omitted since it follows the graphical logic just presented.

Lemma 4: For the case of two-way travel, if knowledge exchange cannot be supported at distance d_1 , then it cannot be supported for any distance $d_2 > d_1$.

We will use the notation D_B as a general representation for the maximum distance across which cooperation can be sustained in the repeated game for the case of bilateral travel. Thus, we can write

$$D_B = \min\{D_B^{T,W;W,NT}, D_B^{NT,W;W,NT}\}.$$

Using this notation, we next present a comparison between D_B and D_C , the distance threshold for the case of travel to a central location.

Proposition 4: For $\gamma > \beta/(N-1)$, $D_B < D_C$ for all δ . For $\gamma < \beta/(N-1)$, $D_B < D_C$ for large enough δ .

Proof: See the Appendix.

An interesting implication of Proposition 4 is that the use of a central location is not necessarily always optimal, despite its advantages in containing travel costs. From this perspective, it is quite natural that industrial clusters exhibit a variety of different configurations, as identified by Markusen (1996). The proposition shows that when complementarities are large, knowledge exchange can be supported across greater distances with travel to a central location rather than bilateral travel. The use of a central location economizes on total travel distance, while the incentives for cheating on the agreement are the same under either structure. When complementarities are small, however, there is a tradeoff: although the use of a central location economizes on travel costs, it also exacerbates firms' incentives to cheat on the agreement. The problem with the central location is that a cheater can take advantage of all other firms at once through a strategy of traveling to the meeting but withholding knowledge. In the case of bilateral travel, the opportunities for cheating are more restricted. A cheater can only take advantage of the firms to which it travels first. The firms that are first to travel to the cheater will learn that the cheater has refused to share

¹⁹We thank an anonymous referee for identifying an error in an earlier draft of the paper, and suggesting that Lemma 4 is likely to be true.

its knowledge, and as a result these firms can reciprocate by withholding their own knowledge when the cheater subsequently visits their facilities. It is impossible to state in general whether the travel gains from a central location outweigh its greater vulnerability to cheating. Nevertheless, Proposition 4 shows that as firms become infinitely patient, the travel cost savings eventually become dominant, since they accrue in every period while the cheating benefits accrue only in the first period of cheating.

Finally, we assess how the distance threshold changes with γ for the case of bilateral travel, as is discussed in the following Proposition.

Proposition 5: For bilateral travel, $\partial D_B / \partial \gamma \geq \partial D_B / \partial \beta > 0$. **Proof:** See the Appendix.

Thus, in the case of bilateral travel, knowledge exchange is sustainable across greater distances when it is more effective in reducing costs, whether knowledge exchange is of the independent type or the complementary type. Furthermore, the distance threshold is more responsive to increases in complementarities than to increases in the value of independent knowledge. These results parallel those for the case of travel to a central location, and reinforce our theme regarding the importance of complementarities in knowledge exchange.

5 Conclusions

Existing theoretical work has paid scant attention to the role of spatial proximity in facilitating knowledge exchange within clusters of technologically interlinked firms. In this paper, we have provided a simple model in which spatial proximity is important due to the need to exchange cost-reducing tacit knowledge via face-to-face contact. We believe our analysis helps to clarify the factors that contribute to the viability of innovation clusters. One factor we highlight is the importance of knowledge complementarities in the sustainability of knowledge-sharing coalitions. Since knowledge exchange is unverifiable, each firm may have incentives to cheat on the other members of its coalition. Nevertheless, we found that even in a one-shot game, knowledge exchange may be an equilibrium if there is sufficient complementarity in the exchange process, that is, if mutual sharing produces a cost reduction beyond what is possible simply through the discrete individual contributions of each party. In the case of repeated trading, firms in our model that cheat on the coalition are excluded from further cooperation, while the remaining members of the coalition continue to cooperate. We find that the presence of greater complementarities facilitates the exchange of tacit knowledge, in the sense that it allows such exchange to be sustained over greater distances.

We also find that the organizational structure of the industry is an important determinant of whether knowledge exchange is viable. We consider two basic structures, one in which all firms travel to a central location to exchange knowledge, and one in which bilateral travel between pairs of firms is necessary. In our model, bilateral travel is necessary when tacit knowledge is embodied in organizational processes and practices such that outsiders must actually visit the relevant facilities of each other firm in order to learn about them. Interestingly, we find that the advantages of particular structures are related to the extent of complementarities in knowledge exchange. When complementarities are large, knowledge exchange is facilitated when firms have the ability to meet in a central location. This may be as a result of sharing a joint research or manufacturing facility, as was the case for the Sematech coalition in the United States or the Taiwan Semiconductor Manufacturing Company. It may also come about because the relevant knowledge is not tied to any physical facilities, and resides independently in the minds of the firm's employees. In this case, it is possible for meetings to take place at any convenient central location, such as a hotel or conference center. When complementarities are large, this structure facilitates knowledge exchange by economizing on travel costs. When complementarities are small, however, a tradeoff emerges in the use of a central location. While the structure reduces travel costs, it is also more vulnerable to cheating than in a structure involving bilateral travel between pairs of firms. With a central location, a firm can opportunistically cheat all other firms in the industry by traveling to the center, passively absorbing knowledge from all its rivals, but withholding its own knowledge. In the bilateral travel structure, however, a firm can only cheat a subset of the other firms in its industry before its cheating behavior is identified and punished. When rival firms visit the cheater before it visits them, rivals learn that the cheater is withholding its knowledge, and reciprocate by withholding their knowledge from it. This makes cheating less attractive than in the central location structure.

The inter-firm trading practices inside geographical clusters are perhaps best documented for the Silicon Valley region. Our analytical structure helps to explain how a cluster such as Silicon Valley maintains inter-firm collaboration among the competing firms given that such collaboration is inherently fragile. The Santa Clara Valley and its surrounding towns of Mountain View, San Jose and Sunnyvale are home to several densely located groups of specialized firms, mainly in the semiconductor industry. For example, as is shown in Angel and Scott (1987), the geographical location of the specialized semiconductor establishments in Silicon Valley displays a close-knit functional distribution among the circuit design establishments, mask-makers, independent test facilities, device assembly houses and other ancillary subcontractors. Such observations provide anecdotal evidence on the significance of spatial proximity in linking clusters of firms with complementary products and technologies. According to Tom Furlong, manager of DEC's work station group in Palo Alto: "An engineering team simply can not work with another engineering team that is three thousand miles away, unless the task is incredibly explicit and well-defined—which they rarely are. If you are not tripping over the guy, you are not working with him, or not working at the level that you optimally could if you co-located."²⁰

Considerable work remains to be done incorporating spatial proximity and knowledge complementarities into economic analysis of knowledge exchange. On the theoretical side, one step would be to include explicitly in our model investments in research and development that generate the knowledge to be exchanged. Another would be to incorporate distance and complementarities in a social network approach, by modeling in detail the formation of links between firms that allow for knowledge sharing, and assessing the equilibrium structure of such networks. A third would be to blend our model of knowledge sharing within coalitions with models of information transmission via job mobility, with the aim of providing a more comprehensive picture of information transmission within clusters. On the empirical side, detailed field studies of collaboration within clusters would be valuable. If documented carefully, they could provide the foundation for econometric research on spatial proximity and knowledge complementarities, and their role in knowledge exchange.

²⁰See Saxenian (1996), p. 157.

Appendix

This appendix contains proofs of all lemmas and propositions. First, however, we present expressions for several derivatives that are used in proofs at various places in the paper. Useful derivatives with respect to γ include

$$\begin{aligned} \frac{\partial \pi^{coop}}{\partial \gamma} &= \frac{2(N-1)}{b(N+1)^2} [a - \widetilde{c} + (\beta + \gamma)(N-1)^2] > 0, \\ \frac{\partial \pi^{ostracize}}{\partial \gamma} &= \frac{-2(N-1)(N-2)}{b(N+1)^2} [a - \widetilde{c}] < 0, \\ \frac{\partial \pi^{W/T}}{\partial \gamma} &= \frac{-2(N-1)(N-2)}{b(N+1)^2} [a - \widetilde{c} + \beta N(N-1)] < 0, \\ \frac{\partial \pi^{T,S;W,NT}}{\partial \gamma} &= \frac{-(N-1)(N-3)}{b(N+1)^2} [a - \widetilde{c} + \frac{(\beta + \gamma)(N-1)^2}{2}] < 0, \end{aligned}$$

and

$$\frac{\partial \pi^{T,W;W,NT}}{\partial \gamma} = \frac{-2(N-1)(N-2)}{b(N+1)^2} [a - \tilde{c} + \frac{\beta N(N-1)}{2}] < 0.$$

Useful derivatives with respect to β include

$$\frac{\partial \pi^{coop}}{\partial \beta} = \frac{2(N-1)}{b(N+1)^2} [a - \tilde{c} + (\beta + \gamma)(N-1)^2] = \frac{\partial \pi^{coop}}{\partial \gamma} > 0,$$
$$\frac{\partial \pi^{ostracize}}{\partial \beta} = \frac{-2(N-1)(N-2)}{b(N+1)^2} [a - \tilde{c}] = \frac{\partial \pi^{ostracize}}{\partial \gamma} < 0,$$
$$\frac{\partial \pi^{W/T}}{\partial \beta} = \frac{4(N-1)}{b(N+1)^2} [a - \tilde{c} + \beta N(N-1)] > 0 > \frac{\partial \pi^{W/T}}{\partial \gamma},$$
$$\frac{\partial \pi^{T,S;W,NT}}{\partial \beta} = \frac{-(N-1)(N-3)}{b(N+1)^2} [a - \tilde{c} + \frac{(\beta + \gamma)(N-1)^2}{2}] < 0,$$

and

$$\frac{\partial \pi^{T,W;W,NT}}{\partial \beta} = \frac{-(N-1)(N-4)}{b(N+1)^2} [a - \tilde{c} + \frac{\beta N(N-1)}{2}] < 0.$$

Lemma 1: Suppose all firms engage in meetings with one another. If knowledge complementarities are small, i.e. $\gamma < \beta/(N-1)$, then the only Nash equilibrium

for the remainder of the stage game is for all firms to withhold their knowledge. If knowledge complementarities are large, i.e. if $\gamma > \beta/(N-1)$, then there are two Nash equilibria, one with no knowledge sharing and one with knowledge sharing by all firms.

Proof: Define $Z = a - \alpha + (\beta + \gamma)(N - 1)$. Then $\pi^{coop} = Z^2/[b(N + 1)^2]$ is the per-firm payoff if all firms cooperate in sharing knowledge. Combining some terms we find that the payoff to a firm that cheats on the cooperative arrangement is $\pi^{cheat} = [Z + (N - 1)(\beta - \gamma(N - 1))]^2/[b(N + 1)^2]$. Then it is easy to show that refusing to share when all other firms share is profitable if and only if $\gamma < \beta/(N - 1)$. In this case, the unique Nash equilibrium in the subgame involves no sharing of knowledge.

If $\gamma > \beta/(N-1)$, then the foregoing logic implies that knowledge sharing is an equilibrium. Nevertheless, there also exists a no-sharing equilibrium. If all firms withhold knowledge, then profits for each firm are $\pi^{withhold} = (a - \alpha)^2/[b(N+1)^2]$. If firm *j* decides to share information when all others withhold, then its cost remains $c_j = \alpha$ while all other firms have cost $c_i = \alpha - \beta$. Thus, firm *j*'s profits are $\pi^{Donate} = [a - N\alpha + (N-1)(\alpha - \beta)]^2/[b(N+1)^2] = [a - \alpha - (N-1)\beta]^2/[b(N+1)^2] < \pi^{Withhold}$. Q.E.D.

Proposition 1: For travel to a central location, knowledge exchange is an equilibrium in the repeated game if

$$d < D_C = \begin{cases} d_C \equiv 2 \left[\pi^{coop} - \pi^{ostracize} \right] & \text{if } \gamma \ge \beta/(N-1) \\ D_C^{W/T} \equiv \frac{2}{\delta} \left[\pi^{coop} - \delta \pi^{ostracize} - (1-\delta)\pi^{W/T} \right] & \text{if } \gamma < \beta/(N-1) \end{cases}$$

Proof: If $\gamma \geq \beta/(N-1)$, inspection reveals that the maximum distance for which cooperation can be supported in the repeated game in this case is just the same as that in the stage game, i.e. $D_C = d_C$. If $\gamma < \beta/(N-1)$, then $V^{coop} > V_C^{W/T}$ if

$$[\pi^{coop} - \frac{d}{2}]/(1-\delta) > \pi^{W/T} - \frac{d}{2} + \delta \frac{\pi^{ostracize}}{1-\delta}$$

or, equivalently, if

$$d < D_C^{W/T} = \frac{2}{\delta} \left[\pi^{coop} - \delta \pi^{ostracize} - (1 - \delta) \pi^{W/T} \right]$$

Proposition 2: For travel to a central location, $\partial D_C/\partial \gamma \geq \partial D_C/\partial \beta > 0$. Furthermore, $\partial D_C/\partial \gamma$ and $\partial D_C/\partial \beta$ are greater for $\gamma < \beta/(N-1)$ than for $\gamma > \beta/(N-1)$.

Proof: Differentiating the expression for D_C shows that

$$\frac{\partial D_C}{\partial \gamma} = \begin{cases} 2\left[\frac{\partial \pi^{coop}}{\partial \gamma} - \frac{\partial \pi^{ostracize}}{\partial \gamma}\right] > 0 & \text{if } \gamma \ge \beta/(N-1) \\ \frac{2}{\delta}\left[\frac{\partial \pi^{coop}}{\partial \gamma} - \delta \frac{\partial \pi^{ostracize}}{\partial \gamma} - (1-\delta) \frac{\partial \pi^{W/T}}{\partial \gamma}\right] > 0 & \text{if } \gamma < \beta/(N-1) \end{cases},$$

where the expressions for the partial derivatives on the right-hand side, along with their signs, are presented at the beginning of the Appendix. Thus, knowledge exchange is sustainable across greater distances when complementarities increase. Furthermore, since it is clear by inspection of the equations at the outset of the Appendix that $d\pi^{W/T}/d\gamma < d\pi^{ostracize}/d\gamma < 0$, and since $\delta \leq 1$, it is easy to show that D_C is concave in γ .

Matters are similar but a bit more complex when we consider comparative statics with regard to β . Differentiating gives

$$\frac{\partial D_C}{\partial \beta} = \begin{cases} 2\left[\frac{\partial \pi^{coop}}{\partial \beta} - \frac{\partial \pi^{ostracize}}{\partial \beta}\right] & \text{if } \gamma \ge \beta/(N-1) \\ \frac{2}{\delta}\left[\frac{\partial \pi^{coop}}{\partial \beta} - \delta\frac{\partial \pi^{ostracize}}{\partial \beta} - (1-\delta)\frac{\partial \pi^{W/T}}{\partial \beta}\right] & \text{if } \gamma < \beta/(N-1) \end{cases}$$

For the case of $\gamma \geq \beta/(N-1)$, a glance at the beginning of the Appendix shows $\partial D_C/\partial\beta = \partial D_C/\partial\gamma > 0$. For $\gamma < \beta/(N-1)$, note that $\partial \pi^{W/T}/\partial\beta > 0$, so the the sign of $\partial D_C/\partial\beta$ is not obvious in general for this case. However, it is easy to see that for the case of large δ , which is our interest, $\partial D_C/\partial\beta > 0$ for $\gamma < \beta/(N-1)$. Since $\partial \pi^{W/T}/\partial\beta > 0 > \partial \pi^{ostracize}/\partial\beta$, it is clear that $\partial D_C/\partial\beta$ is greater for $\gamma < \beta/(N-1)$ than for $\gamma > \beta/(N-1)$. Finally, since $\partial \pi^{W/T}/\partial\beta > 0 > \partial \pi^{W/T}/\partial\gamma$, while $\partial \pi^{coop}/\partial\beta = \partial \pi^{ostracize}/\partial\beta = \partial \pi^{ostracize}/\partial\gamma$, it is clear that $\partial D_C/\partial\gamma \geq \partial D_C/\partial\beta$. Q.E.D.

Lemma 3: In the stage game with bilateral travel, the strategy (T, S; W, NT) can never be optimal.

Proof: Define $R = a - \tilde{c}$, $S = 1/b(N+1)^2$, and $T = (\beta + \gamma)(N-1)^2$. Then $\pi^{ostracize} = R^2/S$, $\pi^{coop} = (R+T)^2/S$, and $\pi^{T,S;W,NT} = (R+T/2)^2/S$. Thus $(\pi^{coop} - K)^2/S$.

 $\pi^{T,S;W,NT}) = (RT + 3T^2/4)/S > (\pi^{T,S;W,NT} - \pi^{ostracize}) = (RT + T^2/4)/S.$ The travel cost required to obtain payoff $\pi^{T,S;W,NT}$ is d(N-1)/2 and the incremental travel cost required to move all the way up to π^{coop} is also d(N-1)/2. Then if $\pi^{T,S;W,NT} - \pi^{ostracize} > d(N-1)/2$ it must also be the case that $\pi^{coop} - \pi^{T,S;W,NT} > d(N-1)/2$. Q.E.D.

Proposition 3 $d_B \leq d_C$.

Proof: If $\gamma < \beta/(N-1)$, then $d_C = d_B = 0$. If $\gamma > \beta/(N-1)$, then Lemma 3 shows that the only relevant comparison is between full cooperation and total non-cooperation. As a result, we simply compare $d_C = 2 \left[\pi^{coop} - \pi^{ostracize}\right] > d_B = \left[\pi^{coop} - \pi^{ostracize}\right]/(N-1)$. Q.E.D.

Proposition 4: For $\gamma > \beta/(N-1)$, $D_B < D_C$ for all δ . For $\gamma < \beta/(N-1)$,

 $D_B < D_C$ for large enough δ .

Proof: For $\gamma > \beta/(N-1)$, the relevant comparison is between full cooperation and total non-cooperation, just as in the stage game. As a result, we simply compare $d_C = 2 \left[\pi^{coop} - \pi^{ostracize} \right] > d_B = \left[\pi^{coop} - \pi^{ostracize} \right]/(N-1).$ For $\gamma < \beta/(N-1)$,

$$D_C = D_C^{W/T} = \frac{2}{\delta} \left[\pi^{coop} - \delta \pi^{ostracize} - (1 - \delta) \pi^{W/T} \right]$$

and

$$D_B = D_B^{T,W;W,NT} = \frac{2}{(N-1)(1+\delta)} \left[\pi^{coop} - \delta \pi^{ostracize} - (1-\delta) \pi^{T,W;W,NT} \right].$$

To compare these expressions, note that $\pi^{W,T} > \pi^{T,W;W,NT}$, since the former involves cheating all (N-1) other firms, while the latter involves cheating only half of them. At the same time, $2/\delta > 2/((N-1)(1+\delta))$ for all N > 2. Thus it is impossible to make a general comparison for all values of δ . Nevertheless, for large enough δ , the bracketed expressions in each distance threshold can be made arbitrarily close together, so $D_B^{T,W;W,NT} < D_C^{W/T}$ because $2/((N-1)(1+\delta)) < 2/\delta$.

Proposition 5: For bilateral travel, $\partial D_B / \partial \gamma \geq \partial D_B / \partial \beta > 0$.

Proof: Since $D_B = \min\{D_B^{T,W;W,NT}, D_B^{NT,W;W,NT}\}$, it is sufficient to show that the derivative on each component of D_B is positive. Differentiating with respect to γ and using the signs established above, we have

$$\frac{\partial D_B^{T,W;W,NT}}{\partial \gamma} = \frac{2}{(N-1)(1-\delta)} \left[\frac{\partial \pi^{coop}}{\partial \gamma} - \delta \frac{\partial \pi^{ostracize}}{\partial \gamma} - (1-\delta) \frac{\partial \pi^{T,W;W,NT}}{\partial \gamma} \right] > 0,$$

and

$$\frac{\partial D_B^{NT,W;W,NT}}{\partial \gamma} = \frac{1}{(N-1)} \left[\frac{\partial \pi^{coop}}{\partial \gamma} - \frac{\partial \pi^{ostracize}}{\partial \gamma} \right] > 0.$$

Differentiating with respect to β yields very similar results. Indeed, $\partial D_B^{NT,W;W,NT} / \partial \gamma = \partial D_B^{NT,W;W,NT} / \partial \beta$. However, the results at the outset of the Appendix show $\partial \pi^{T,W;W,NT} / \partial \gamma < \partial \pi^{T,W;W,NT} / \partial \beta < 0$, so $\partial D_B^{T,W;W,NT} / \partial \gamma > \partial D_B^{T,W;W,NT} / \partial \beta$.

Q.E.D.

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Figure 1: Distance Threshold and Complementarities

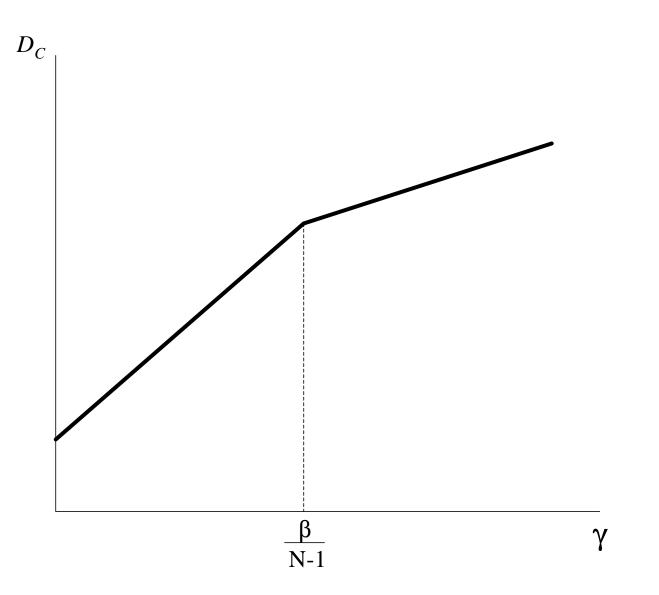


Figure 2A: Distance and Cooperation with Bilateral Travel $\gamma < \beta/(N-1)$ VVcoop Vostracize $V^{T,W;W,NT}$ d_C $d^{T/W}$ d $\overline{2(N-1)}$ $D_B^{T,W;W,NT}$

